STAT /MATH 3360

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Time and Place: Tues, Thurs 13:05 – 14:25
Life Scs. Buildg.

Text Book: A First Course in Probability by Sheldon Ross
7th Edition,

1. Combinatorial Analysis
2. Axioms of Probability
3. Conditional Probability and Independence
4. Random Variables, Expectation, Functions of a Random Variables,
5. Distribution of Discrete Random Variables
6. Distribution of Continuous Random Variables
7. Jointly Distributed Random Variables
8. Properties of Expectation, Covariance, Conditional Expectation, Moments Generating Function,
9. Limit Theorems

Marking:

Assignments	25%
Midterm	25%
Final Exam	50%

Goal: To develop an understanding of the principles and techniques of Probability.
To Learn how to use probability to model various situations.
To learn basic theoretical results essential for statistics.

STAT/MATH 3360
FINAL EXAM

Dec. 13, 2005
12:00 - 3:00pm

1. Let X denote the length in minutes for a long distance telephone conversation. The pdf of X is $f(x) = \frac{1}{10} e^{-x/10}; x > 0$

   (a) find the moment generating function $M_x(t)$.
   (b) use $M_x(t)$ to find the average length of a call.
   (c) find the variance of X
   (d) what is the probability that the length of a call will be less than 2 minutes.
   (e) find the distribution of $Y = -\frac{1}{10} \log X$

2. Let X denote the temperature (°C) and let Y denote the time in minutes that it takes an auto to get ready to start. Assume the point pdf. $(x, y) is$

   $$f(x, y) = c(4x + 2y + 1); \quad 0 \le x \le 40, \ 0 \le y \le 2$$

   (a) find the value of c that makes it pdf
   (b) find the probability that on a randomly selected day, the air temperature will exceed 20°C and it will take at least one minute for the car to be ready to start.
   (c) find the marginal density of X.

3. Two urns, I and II contain respectively 2 white and 1 black balls, 1 white and 5 blackballs. One ball is transferred from the term I to II and then one ball is drawn from the latter. It happens to be white. What is the probability that the transferred ball was black.

4. Suppose that successive births are independent, and the probability of a male birth is p. Under each of the following conditions, find the probability that there are two boys and two girls in a family of four:

(a) no further information is available;
(b) the eldest child is known to be a son;
(c) the eldest and youngest are known to be a son and a daughter, respectively;
(d) it is known that there is at least one son and at least one daughter.

5. Ten mice are randomly selected and divided into two groups of five. Each mouse in the first group is given a dose of a certain poison which is sufficient to kill one in ten; each mouse in the second group is given a dose sufficient to kill three in ten. What is the probability that there will be more deaths in the first group than in the second?

6. Let X and Y are independent Poisson Variables with parameter λ and μ i.e.

$$f(x) = e^{-\lambda}\frac{\lambda^x}{x!}; \qquad x = 0,1,2,\ldots \text{ and}$$

$$f(y) = e^{-\mu}\frac{\mu^y}{y!}; \qquad y = 0,1,2\ldots$$

Show that the conditional distribution of X given X+Y is binomial.

7. Let X and Y are independent Gamma variables

$$f(x) = \frac{1}{\Gamma(m)}e^{-x}x^{m-1}; x \geq 0 \text{ and} \qquad f(y) = \frac{1}{\Gamma(n)}e^{-y}y^{n-1}; y \geq 0$$

show that $u = x + y$ and $v = \frac{x}{y}$ are independent.