Stat/Math 4066/5066 - Fall 2006
Advanced Statistical Theory
Course Outline

Instructor:	Dr. E. Susko (Chase 115)
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Time:	TR 1:00-2:30
Place:	Chase 319
Office hours:	TR 9:30-11:30

Marking Scheme

Assignments	50%
Midterm	20%
Final exam	30%

Text:

Mathematical Statistics: Basic Ideas and Selected Topics, Peter J. Bickel and Kjell A. Doksum, 2nd Edition, Prentice Hall.

Topics

Probability and Distributional Theory	Appendices A,B
Statistical Models, Goals and Performance Criteria	Ch 1
Methods of Estimation	Ch 2
Measures of Performance	Ch 3
Testing and Confidence Bounds	Ch 4
Asymptotic Approximations	Ch 5
Inference in The Multiparameter Case	Ch 6

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Stat/Math 4066 - Intermediate Statistical Theory - Winter 2006
Final Examination

Each question is equally weighted

1. (a) Suppose that, independently, $X_1, \ldots, X_n$ have density

$$p(x; \mu, \sigma) = \sigma^{-1} \exp\{-(\frac{x-\mu}{\sigma})\}, \qquad x \geq \mu$$

   i. Find a one-dimensional sufficient statistic for μ when σ is known.
   ii. Find a two-dimensional sufficient statistic for (μ, σ) when both are unknown.

   (b) Suppose that, independently, $X_1, \ldots, X_n$ have a $N(\mu, \sigma^2)$ distribution and that $Y_1, \ldots, Y_m$ have a $N(\eta, \tau^2)$ distribution. Find minimal sufficient statistics when $\sigma = \tau$, μ, η and σ are arbitrary.

   (c) Suppose that, independently, $X_1, \ldots, X_n$ come from a location model: $p(x; \theta) = f(x - \theta)$, where f is a probability density function. Give an argument indicating that $S^2 = \sum_i (x_i - \bar{x})^2$ is ancillary for θ. Given that this is the case, how should inference be conducted? Why would this not matter if f was the standard normal density?

2. Which of the following are exponential families. Give reasons for your answer.

   (a) $p(x; \theta) = \binom{n+x-1}{x}(1-\theta)^x \theta, \qquad x = 0, 1, \ldots$

   (b) $p(x; \theta) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1}(1-x)^{s-1}, 0 < x < 1, \ \theta = (r, s)$

   (c) $p(x; \theta) = \frac{x^{\alpha-1}}{\Gamma(\alpha)\beta^\alpha} \exp[-x/\beta], \qquad x > 0, \ \theta = (\alpha, \beta) > 0$

   (d) The $N(\theta, \theta^2)$ distribution

3. Suppose that, independently, $X_1, \ldots, X_n$ have probability mass function

$$p(x; \theta) = (1-\theta)\theta^x, \qquad x = 0, 1, \ldots$$

   (a) Find the ML estimator of θ.

   (b) Find the ML estimator of $\eta = \log[\theta/(1-\theta)]$.

   (c) What is the expected information matrix for θ? [For full marks all expectations should be derived].

   (d) Suppose now a Bayesian framework with prior

$$\pi(\theta) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} \theta^{r-1}(1-\theta)^{s-1}$$

   Find the Bayes estimator of θ^k under squared error loss.

4. Suppose that, given $X_1, \ldots, X_n$, $Y_1, \ldots, Y_n$ are independent Poisson random variables with $E[Y_i] = \exp\{\beta_0 + \beta_1 X_i\}$. Suppose the X_i are independent and from a $N(\mu, \sigma^2)$ distribution.

   (a) Explain why it is appropriate to treat $X_1, \ldots, X_n$ as if they are fixed, much like you would in a regression model.

   (b) Write the model in two-parameter exponential form and give a two-dimensional sufficient statistic.

   (c) Find a lower bound on the variances of unbiased estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ of β_0 and β_1.

5. Suppose that, independently, $X_1, \ldots, X_n$ have probability density function

$$p(x; \theta) = \theta x^{\theta-1}, 0 < x < 1$$

(a) Find the MLE of $1/\theta$. Is it unbiased? Does it achieve the information inequality lower bound?

(b) Show that $\bar{X}$ is an unbiased estimator of $\theta/(\theta+1)$. Does $\bar{X}$ achieve the information inequality lower bound?

6. Suppose that, independently, $X_1, \ldots, X_n$ have probability density function

$$p(x;\theta) = cx^{c-1}\exp[-x^c/\theta]/\theta, \quad x > 0$$

where c is a fixed known constant.

(a) Derive the UMP α-level test of $H_0 : \theta \leq \theta_0$ against $H_A : \theta > \theta_0$.

(b) Give an expression for the power function of the test that involves the cumulative distribution function of a χ^2_{2n} random variable.

7. Suppose that, independently, $X_1, \ldots, X_n$ come from a $N(0, \sigma^2)$ distribution. Find the uniformly most accurate $(1-\alpha) \times 100\%$ lower bound for σ. Justify your answer.