Statistics 4300/5300 - Fall 2003
Stochastic Processes
Course Outline

Instructor:	Dr. E. Susko (Chase 115)
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Time:	TR 1:00-2:30
Place:	Chase 312 (Colloquium Room)
Office hours:	TR 10-12
Text:	Introduction to Probability Models (8th Edition)
	Sheldon M. Ross

Marking Scheme

Assignments	35%
Midterm	25% Thurs, October 16 7-8:30 Coll. Room
Final exam	40%

Intended Material

1. Basic Probability and Conditioning
2. Branching Processes
3. Poisson Processes
4. Discrete Time Markov Chains
5. Continuous Time Markov Chains
6. Additional Topics

There will be no supplemental examination in this course.

Statistics 4300/5300 - Stochastic Processes - Fall 2003
Final Exam

- Each portion of a question is worth 5 marks.
- Student(s) in sta4300 will have their worst two parts of a question dropped. For instance, if 1 (b) and 3 (c) are not completed, the exam will be marked as if they were not present.

1. Individuals arrive in a fixed region according to a Poisson process with rate λ. Independently, each individual gives birth to i children with probability $p_i \propto a^i$, $i = 0, 1, \ldots$.
   (a) What is the probability that no children are born in the region?
   (b) What is the expected population size of children and arrivals?

2. Calculate the following for a Poisson process with rate λ.
   (a) $P(N(t) = 1 | N(3t) = 5)$.
   (b) $\mathrm{Var}(T_1 | N(2) = 1)$ where T_1 is the time to the first event.
   (c) $E[S_2]$ and $E[S_1 S_2]$ where S_1 and S_2 are the waiting times for the first and second events.
   (d) $\mathrm{Cov}[N(t+s), N(t)]$.

3. Consider a branching process where each individual currently in the population always gives birth to an even number of offspring. Suppose that the offspring distribution is

$$P(Z_n = 2j) = p^j(1-p), \qquad j = 0, \ldots$$

   (a) Obtain the probability that the population is extinct by the third generation when $p = 1/3$.
   (b) Obtain the probability that the population goes extinct at the third generation when $p = 1/3$. That is, the probability that the population is not extinct in the second generation but is extinct in the third generation.
   (c) Obtain the mean population size at the nth generation. Give a general expression.

4. Suppose that a discrete time Markov chain with state space $S = \{0, 1, 2, 3, 4\}$ has transition probability matrix

$$P = \begin{pmatrix} 0 & 2/3 & 0 & 0 & 1/3 \\ 4/7 & 0 & 3/7 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1/8 & 7/8 \end{pmatrix}$$

   (a) Decompose the state space of the Markov chain into closed communicating classes and transient states.
   (b) Determine the limits, as $n \to \infty$ of $p_{43}^{(n)}$ and $p_{44}^{(n)}$.
   (c) Determine the absorption probabilities from the transient state(s) to the closed communicating class(es).
   (d) Determine $p_{31}^{(n)}$ and the expected time between transitions to state 4.

5. The travel of a business man is being monitored. The traveller has business in Halifax, Toronto and Vancouver. Given that he is in Halifax, he travels to Toronto next. Given that he is in Toronto his next trip out will be to Halifax with probability 1/3. When he is in Vancouver his next trip out is equally likely to be to Toronto or Halifax. Independently, the periods that he remains in Halifax, Toronto and Vancouver are exponentially distributed with means of 2, 3 and 5 days respectively. Let $X(t)$ be the location of the man at time t with $0, 1$ and 2 denoting Halifax, Toronto and Vancouver respectively.

   (a) Derive the rate matrix for the Markov chain.

   (b) Give an equation that relates the $p'_{10}(t)$ to the $p_{ij}(t)$. You need not solve it.

   (c) Calculate the limiting probabilities: $\lim_{t\to\infty} p_{ij}(t)$.

   (d) Is this an irreducible continuous-time Markov chain? Justify your answer.

   (e) If the man is equally likely to be in any of the three cities at time 0, calculate the probability that the third city he visits is Halifax.

6. People line up in for service at a single grocery clerk according to a Poisson process with rate λ. The times required for service are exponentially distributed with rate μ. Let $N(t)$ denote the number of individuals waiting in line.

   (a) Derive the rate matrix for the $\{N(t)\}$ process. Justify your answer.

   (b) Give conditions under which the limiting probabilities will exist and calculate them under these conditions.

   (c) Is this a time reversible Markov chain? Give reasons for your answer.