STAT 3340 Assignment 4, Fall 2026, due Monday, November 16, 11:59 PM

1. The regression model $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$ was fit to a data set with $n = 24$ observations, and resulted in MSE=16, with $\hat{\beta}$ and $(X'X)^{-1}$ given by:

```
Mhatbeta= c(1000, 100, -200)
xpxinverse=matrix(c(15,8,7,8,12,5,7,5,20), byrow=T,ncol=3)
```

- 1a) Construct a 95% confidence interval for β_1.
- 1b) Construct simultaneous 90% confidence intervals for β_1 and β_2.
- 1c) Construct a 99% confidence interval for $\beta_0 - 2\beta_1 + \beta_2$.
- 1d) Is the value $\beta = c(950, 90, -230)$ contained in the 95% confidence ellipse for β? (Hint: Calculate the value of $(\hat{\beta} - \beta)^T (X'X)(\hat{\beta} - \beta)$ and compare it to $pMSE\ qf(.95, p, n - p)$.) Show your work.

2. The following calculates the SSE and MSE for the model $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \epsilon$, for the "cement" data set.

```
data=read.csv("http://www.mathstat.dal.ca/~bsmith/stat3340/Data/cement.csv")
  #read data
 X=as.matrix(cbind(rep(1,13),data[,-c(1,2)])) #construct the X matrix
 y=matrix(data[,2],byrow=T,ncol=1)   #construct Y
  n=dim(X)[1]   #number of observations
  k=dim(X)[2]-1 #number of predictor variables in full model

beta=solve(t(X)%*%X)%*%t(X)%*%y  #calculate least squares estimates
SSEfull=t(y)%*%y-t(beta)%*%t(X)%*%y  #get the error SS for full model
MSEfull=SSEfull/(n-k-1)                    #MSE under full model
```

Answer the following questions by computing SSE for the appropriate reduced models using matrix operations, then calculating the F statistic for the partial F test, and calculating the p-value using the "pf" command. (You can check your results using the builtin "anova" function.)

- 2a) Test the hypothesis that $\beta_1 = 0$.

```
#Xred=X[,???] construct the appropriate X matrix for the reduced model
#betared=solve(t(Xred)%*%Xred)%*%t(Xred)%*%y  #calculate LSE for reduced model
#SSEred=t(y)%*%y-t(betared)%*%t(Xred)%*%y   #get the SSE for the reduced model
#Fobs=((?-?)/?)/?  # calculate the F statistic
#Fobs # print the observed F
#pv=1-pf(Fobs,?,?) #calculate the p-value
#pv #print the p-value
```

- 2b) Test the hypothesis that $\beta_1 = \beta_2 = 0$.

```
# enter your R commnds here
```

- 2c) Test the hypothesis that $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$.

```
# enter your R commnds here
```

3. Prove that the matrices $I - J/n$, and $H - J/n$ are orthogonal to one another. You can use the fact that $HX = X$.