ACSC/STAT 3703 - Winter 2026 - Assignment 3, due Wednesday, Feb 25

1. For any discrete random variable X with moment generating function $M_X(z)$ and probability generating function $P_X(z)$
   (a) Show that $P_X(z) = M_X(log_e(z))$.
   (b) Using the fact that $E[X^k] = M_X^{(k)}(0)$, show that $P^{'}(1) = E[X]$ and $P^{''}(1) = E[X(X-1)]$.
2. (a) where X has pmf $p_k = (\frac{\beta}{1+\beta})^k \frac{1}{1+\beta}$, $k = 0, 1, 2, \ldots$, show that $P_X(z) = (1 - \beta(z-1))^{-1}$
   (b) Suppose that $X_1, X_2, \ldots, X_r$ are independent, identically distributed with the pmf given in part (a). Derive the probability generating function of $N = \sum_{j=1}^{r} X_j$.
   (c) Using the fact that $E(N) = P_N^{'}(1)$ and $E(N(N-1)) = P_N^{''}(1)$, derive the mean and variance of N.
3. For the Poisson distribution with mean λ, derive the values of a and b for the (a,b,0) recursion by examining $\frac{p_k}{p_{k-1}}$.
4. Using the values of a and b which you just calculated, letting $\lambda = 1$, and approximating $p_0 = e^{-1}$ as .368,
   (a) use the (a,b,0) recursion to calculate p_1, p_2, p_3.
   (b) now consider the associate 0-modified Poisson with $p_0^M = 0.10$. Calculate the alues of p_1^M, p_2^M, p_3^M
5. Problem 6.4. Show that for the ETNB with $\beta > 0$, $r > -1$ but $r \neq 0$, the values of p_k given by the (a,b,1) recursion are positive and $\sum_{k=1}^{\infty} p_k < \infty$.
6. Suppose you have a sample of size n from a discrete distribution on $0, 1, 2, \ldots$, and in the sample there are n_0 values equal to 0, n_1 equal to 1, and so on. You make a plot of $k\frac{n_k}{n_{k-1}}$ vs k and observe that the slope of the plot is close to 1. Does this suggest that the underlying distribution is binomial, negative binomial or Poisson?