Combinatorial Algebra meets Algebraic Combinatorics

Seventeenth Annual Meeting
Dalhousie University, Halifax, Canada

FRIDAY JANUARY 24 (Studley Campus, Chase 319)

2 – 2:30		REGISTRATION
2:30 – 2:50	Brown	*Commutative Algebra, Colourings and Network Reliability*
3 – 3:20	DeGagne	*On Reliability Roots of Simplicial Complexes and Matroids*
3:30 – 3:50	Milson	*Composition sum identities and the distribution of coordinate values in a simplex*
4 – 4:30		COFFEE
4:30 – 5:20	Francisco	*Studying monomial ideals via posets*
5:30 – 5:50	Bhat	*Properties of some classes of generalized Borel ideals*
6:00 – 6:20	Ahmad	*On different classes of Monomial Ideals associated to lcm-lattices*
		INFORMAL DINNER

SATURDAY JANUARY 25 (Sexton Campus, Building B (J052), Room B311)

9 – 9:30	Iarrobino	*Jordan type and Artinian Gorenstein algebras*
9:40 – 10:00	Altafi	*Jordan types of Artinian algebras with height two*
10:10 – 10:30	Nicklasson	*Subalgebras of a polynomial ring with minimal Hilbert function*
10:30 – 11:00		COFFEE
11:00 – 11:50	Miller	*Minimal resolutions of monomial ideals*
12 – 12:20	Ordog	*Lattice paths from Koszul double complexes*
12:30 – 2:30		LUNCH
2:30 – 3:20	Yu	*Higher Connectivity of Tropicalizations*
3:30 – 3:50	Hodges	*A non-iterative formula for straightening fillings of Young diagrams*
4 – 4:30		COFFEE
4:30 – 4:50	Ashraf	*Tutte coefficients as tropical intersection numbers*
5 – 5:20	Galetto	*Symmetric shifted ideals*
5:30 – 5:50	Jayanthan	*Regularity of powers of quadratic sequences and binomial edge ideals*
7:00 –		CONFERENCE DINNER [LA FRASCA, 5650 SPRING GARDEN ROAD]

SUNDAY JANUARY 26 (Sexton Campus, Building B (J052), Room B308)

9 – 9:30	Bergeron	*New $GL_\infty \times S_n$-modules of diagonal harmonic polynomials*
9:40 – 10:00	Martinez	*Geometric Constructions for Lie Algebra Type A_3*
10 – 10:30		COFFEE
10:30 – 11:20	Van Willigenburg	*The e-positivity of chromatic symmetric functions*
11:30 – 12	Aliniaeifard	*A categorification of the Malvenuto-Reutenauer algebra via a tower of groups*

Abstracts:

1. **Speaker:** Jason Brown (*Dalhousie University*)

   Title: Commutative Algebra, Colourings and Network Reliability

   Abstract: A k-colouring of a graph G is an assignment of a colour from the set $\{1, 2, ..., k\}$ to each vertex in the graph so that the two ends of every edge has different colours; the chromatic polynomial of G enumerates, for each k, the number of k-colourings the graph has. On the other hand, if we envision that every vertex of G is operational, but the edges fail independently with probability q, then the all-terminal reliability of G is the probability that all the vertices can communicate. These two disparate problems have a common thread that is filled in via commutative algebra. In this talk we'll describe the connection, and show how it can be used to shed new combinatorial light on the two graph theory problems. As well, some tantalizing related open problems on Grobner bases are waiting at the surface.

2. **Speaker:** Corey DeGagne (*Dalhousie University*)

   Title: On Reliability Roots of Simplicial Complexes and Matroids

   Abstract: Assume that the vertices of a graph G are always operational, but the edges of G fail independently with probability $q \in [0, 1]$. The all-terminal reliability of G is the probability that the resulting subgraph is connected. The all-terminal reliability is a polynomial in q, and it was conjectured that all the roots of (nonzero) reliability polynomials fall inside the closed unit disk. It has since been shown that there exist some connected graphs which have their reliability roots outside the closed unit disk, but these examples seem to be few and far between, and the roots are only barely outside the disk. In this talk we generalize the notion of reliability to simplicial complexes and matroids and investigate when, for small simplicial complexes and matroids, the roots fall inside the closed unit disk.

3. **Speaker:** Rob Milson (*Dalhousie University*)

   Title: Composition sum identities and the distribution of coordinate values in a simplex

   Abstract: Utilizing spectral residues of parameterized, recursively defined sequences, we develop a general method for generating identities of composition sums. Specific results are obtained by focusing on coefficient sequences of solutions of first and second order, ordinary, linear differential equations. Regarding the first class, the corresponding identities amount to a proof of the exponential formula of labelled counting. The identities in the second class can be used to establish certain geometric properties of the simplex of bounded, ordered, integer tuples.

4. **Speaker:** Chris Francisco (*Oklahoma State University*)

   Title: Studying monomial ideals via posets

   Abstract: Monomial ideals provide an interesting bridge between commutative algebra and combinatorics. We survey some work that uses the structure of posets to study properties of monomial ideals, focusing in particular on efforts to extend the usual notion of a Borel monomial ideal. Some of this is joint work with Mermin and Schweig; we will also discuss results of Bhat, Cook II, and others, as well as some open questions.

5. **Speaker:** Ashwini Bhat (*University of Oklahoma*)

   Title: Properties of some classes of generalized Borel ideals

   Abstract: Borel ideals are a thoroughly understood class of monomial ideals known to be minimally resolved by the Eliahou-Kervaire resolution. In 2013, Francisco, Mermin, and Schweig introduced Q-Borel ideals, a generalization of Borel ideals defined by relations in a poset. We extend their work, using combinatorial methods to describe some homological properties of these ideals.

6. **Speaker:** Sarfraz Ahmad (*COMSATS University Islamabad(Lahore Campus), Lahore-Pakistan*)

   Title: On different classes of Monomial Ideals associated to lcm-lattices

   Abstract: Let K be a field $S = K[x_1, ..., x_n]$ be the polynomial ring in n numbers of variables. To each monomial ideal I in S, one can associate its lcm-lattice denoted by $L(I)$. Consider I and J be two monomial ideals such that, $L(I)$ is isomorphic to $L(J)$. In this talk we discuss different classes of above mentioned ideals such that $L(I^n)$ and $L(J^n)$ are isomorphic for different values of n. In the end we state few conjectures.

7. **Speaker:** Tony Iarrobino, (*Northeastern University*)

   Title: Jordan type and Artinian Gorenstein algebras

   Abstract: Let ℓ be an element in the maximal ideal of a local Artinian algebra A. Recall that the Jordan type P_ℓ of the multiplication map $m_\ell : A \to A$ is the partition giving the sizes in the Jordan block decomposition for m_ℓ, which is nilpotent. The Jordan type in general gives more information than whether ℓ is weak or strong Lefschetz. The Jordan degree type is a finer invariant, which specifies the initial degrees of "strings" comprising P_ℓ.

   When A is graded Gorenstein, results of T. Harima and J. Watanabe concerning central simple modules show that there is a reflexive symmetry in the Jordan degree type. This has been exploited by B. Costa and R. Gondim to prove that, given the Hilbert function of A, there are restrictions on the Jordan type. We outline this symmetry and give examples.

   When A is non-graded Gorenstein, then the associated graded algebra of A has a symmetric decomposition; this, combined with the deformation properties of Jordan type leads to families Gor(H) (Gorenstein algebras of Hilbert function H) in embedding dimension three having several irreducible components. This is joint work with Pedro Macias Marques.

8. **Speaker:** Nasrin Altafi (*KTH Stockholm*)

   Title: Jordan types of Artinian algebras with height two

   Abstract: Multiplication by a linear form ℓ on a Artinian algebra A determines a nilpotent linear operator on A, the Jordan type of this operator is an integer partition of the dimension of A as a vector space. The weak Lefschetz and the strong Lefschetz properties of A can be determined from the Jordan type of a generic ℓ of A.

   In this talk, we determine which partitions of n may occur as the Jordan type for some linear form ℓ on a graded complete intersection Artinian quotient $A = R/(f, g)$ of the polynomial ring R in two variables. We explain how we obtain such partitions algebraically and combinatorially as well as describing their connection.

   We then briefly explain for a fixed partition P how the combinatorial properties of P could be applied to determine the minimal number of generators of a generic height two Artinian algebra with P as its Jordan type for some linear form. This is a joint work with A. Iarrobino, L. Khatami and J. Yaméogo.

9. **Speaker:** Lisa Nicklasson (*Stockholm University*)

   Title: Subalgebras of a polynomial ring with minimal Hilbert function

   Abstract: In a recent paper by Boij and Conca the upper and lower bounds for the Hilbert function of subalgebras of a polynomial ring are discussed. In this talk we will study subalgebras generated in degree two with minimal Hilbert function. These subalgebras are generated by strongly stable sets of monomials. To minimize the Hilbert function we want to firstly minimize the numbers of variables, and secondly the multiplicity of the algebra. This boils down to a purely combinatorial problem, as the multiplicity can be computed by counting the number of maximal north-east lattice paths in an diagram representing the strongly stable set.

10. **Speaker:** Ezra Miller (*Duke University*)

    Title: Minimal resolutions of monomial ideals

    Abstract: It has been an open problem since the 1960s to construct closed-form, canonical, combinatorial minimal free resolutions of arbitrary monomial ideals in polynomial rings. This talk explains how to solve the problem, in characteristic 0 and almost all positive characteristics, using sums over lattice paths of combinatorial data from simplicial complexes, one simplicial complex for each lattice point. Any minimal free resolution of any monomial ideal must – either implicitly or explicitly – produce homomorphisms between various homology groups of these simplicial complexes. Therefore an important aspect of the solution is an explicit way to write down canonical homomorphisms between these homology groups without choosing bases. Joint work with Jack Eagon and Erika Ordog.

11. **Speaker:** Erika Ordog (*Duke University*)

    Title: Lattice paths from Koszul double complexes

    Abstract: It has been an open problem since the 1960s to construct closed-form, canonical, combinatorial minimal free resolutions of arbitrary monomial ideals in polynomial rings. Abstractly, our solution constructs resolutions by splitting a deceptively simple double complex whose rows and columns are all Koszul complexes. The algebra of spectral sequences for this Koszul double complex translates into combinatorics of "chain-link fences", which reflect matroidal data – higher-dimensional versions of spanning tree information – descending along lattice paths. Joint work with Jack Eagon and Ezra Miller.

12. **Speaker:** Josephine Yu (*Georgia Institute of Technology*)

    Title: Higher Connectivity of Tropicalizations

    Abstract: The tropicalization of an irreducible variety is a polyhedral complex. The structure theorem stays that the polyhedral complex is balanced and is connected through codimension one. I will give an introduction to tropical geometry and discuss a recent joint work with Diane Maclagan where we prove a stronger version of the structure theorem — the tropicalization is connected through codimension one even after removing some maximal faces. Since (cones over) skeleta of polytopes are tropicalizations of irreducible varieties, this result gives a generalization of Balinskis theorem for higher dimensional skeleta of polytopes. For the proof, we use a tropical version of Bertini's theorem, which says that the intersection of the tropicalization of an irreducible variety and a generic hyperplane is again the tropicalization of an irreducible variety.

13. **Speaker:** Reuven Hodges (*University of Illinois Urbana-Champaign*)

    Title: A non-iterative formula for straightening fillings of Young diagrams

    Abstract: Young diagrams are fundamental combinatorial objects in representation theory and algebraic geometry. Many constructions that rely on these objects depend on variations of a straightening process that expresses a filling of a Young diagram as a sum of semistandard tableaux subject to certain relations. It has been a long-standing open problem to give a non-iterative formula for this straightening process. In this talk I will give such a formula. I will then use this non-iterative formula to prove that the coefficient of the leading term in the straightening is either 1 or -1, generalizing a theorem of Gonciulea and Lakshmibai.

14. **Speaker:** Ahmed Umer Ashraf (*Western University*)

    Title: Tutte coefficients as tropical intersection numbers

    Abstract: Recently Gioan and Las Vergnas gave an expression for the coefficients of Tutte polynomial as a sum of products of beta invariants of certain minors. Similar coefficients occur in the CSM class of matroids as defined by de Medrano, Rincon and Shaw. We provide an expression for coefficient of Tutte polynomial as tropical intersection numbers, using Gioan-Las Vergnas expression of Tutte polynomial. This work is in collaboration with Spencer Backman.

15. **Speaker:** Federico Galetto (*Cleveland State University*)

    Title: Symmetric shifted ideals

    Abstract: The defining ideals of star configurations are interesting objects that live at the intersection of algebra, geometry, and combinatorics. Until recently, little was known about their symbolic powers. We approach this subject by introducing the class of symmetric shifted ideals. These are monomial ideals that are stable under the action of the symmetric group permuting the variables. In this talk, I will introduce symmetric shifted ideals and present results about their Betti numbers with interesting consequences for symbolic powers of star configurations.

16. **Speaker:** A. V. Jayanthan (*IIT Madras*)

    Title: Regularity of powers of quadratic sequences and binomial edge ideals

    Abstract: This talks is on a joint work with Arvind Kumar and Rajib Sarkar. In this work, we obtain an upper bound for the Castelnuovo-Mumford regularity of powers of ideals generated by homogeneous quadratic sequence in a polynomial ring in terms of the regularity of its related ideals and degrees of its generators. As a consequence, we compute upper bound for the regularity of powers of binomial edge ideal, J_G, of some classes of graphs. We generalize a result of Matsuda and Murai to show that the regularity of J_G^s is bounded below by $2s + \ell(G) - 1$ for all $s \geq 1$, where $\ell(G)$ is the length of a longest induced path in G. Using these two bounds we compute explicitly the regularity of powers of binomial edge ideals of cycle graphs, star graphs and balloon graphs. Also, we give a sharp bounds for the regularity of powers of almost complete intersection binomial edge ideals.

17. **Speaker:** Francois Bergeron (*Université du Québec à Montréal*)

    Title: New $GL_\infty \times S_n$-modules of diagonal harmonic polynomials

    Abstract: I propose new modules that explain and expand a conjecture about Schur positivity that I formulated 25 years ago. One interesting aspect is that this furnishes natural links between questions that have risen recently in several areas.

18. **Speaker:** Lucy Martinez (*Stockton University, NJ*)

    Title: Geometric Constructions for Lie Algebra Type A_3

    Abstract: Kostants weight multiplicity formula is an alternating sum over the symmetry group of a Lie algebras root system, known as the Weyl group. This formula is used to compute the multiplicity of a weight in an irreducible highest weight representation of a Lie algebra. In this research, we focus our attention to describing the Weyl alternation sets, which consist of subsets of the Weyl group elements that contribute nontrivially to Kostants weight multiplicity. From this, we present geometric constructions for these sets combinatorially and geometrically.

19. **Speaker:** Stephanie van Willigenburg (*University of British Columbia*)

    Title: The e-positivity of chromatic symmetric functions

    Abstract: The chromatic polynomial was generalized to the chromatic symmetric function by Stanley in his seminal 1995 paper. This function is currently experiencing a flourishing renaissance, in particular the study of the positivity of chromatic symmetric functions when expanded into the basis of elementary symmetric functions, that is, e-positivity.

    In this talk we approach the question of e-positivity from various angles. Most pertinently we resolve the 1995 statement of Stanley that no known graph exists that is not contractible to the claw, and whose chromatic symmetric function is not e-positive.

    This is joint work with Soojin Cho, Samantha Dahlberg, Angele Foley and Adrian She, and no prior knowledge is assumed.

20. **Speaker:** Farid Aliniaeifard (*University of British Columbia*)

    Title: A categorification of the Malvenuto-Reutenauer algebra via a tower of groups (with Nat Thiem)

    Abstract: There is a long tradition of categorifying combinatorial Hopf algebras by the modules of a tower of algebras (or even better via the representation theory of a tower of groups). From the point of view of combinatorics, such a categorification supplies canonical bases, inner products, and a natural avenue to prove positivity results. Recent ideas in supercharacter theory have made fashioning the representation theory of a tower of groups into a Hopf structure more tractable. As a demonstration, this talk reports on the results of the following challenge: (1) Pick a well-known combinatorial Hopf algebra, (2) Find a way to categorify the structure via a tower of groups. In this case, we show how to find the Malvenuto Reutenauer Hopf algebra in the representation theory of a tower of elementary abelian p-groups.