DALHOUSIE MATHEMATICS COLLOQUIUM

Thursday May 17, 2:30 pm, Chase 319

Speaker: Maxim Burke

(University of Prince Edward Island)

Approximation by entire functions in the construction of order-isomorphisms and large cross-sections

A theorem of Hoischen states that given $\varepsilon > 0$ and a closed discrete set $T \subseteq \mathbb{R}^t$, any C^n function $g : \mathbb{R}^t \to \mathbb{R}$ can be approximated by an entire function f so that for $k = 1, \ldots, n$, for all $x \in \mathbb{R}^t$ and for each multi-index α such that $|\alpha| \leq n$,

(a) $|(D^\alpha f)(x) - (D^\alpha g)(x)| < \varepsilon(x)$;

(b) $(D^\alpha f)(x) = (D^\alpha g)(x)$ if $x \in T$.

This theorem has been useful in helping to analyze the existence of entire order-isomorphisms of everywhere non-meager subsets of $\mathbb{R}$, analogous to the Barth-Schneider theorem, which gives entire order-isomorphisms of countable dense sets, and the existence for a given everywhere non-meager subsets A of $\mathbb{R}^{t+1} \cong \mathbb{R}^t \times \mathbb{R}$, of entire functions f so that $\{x \in \mathbb{R}^t : (x, f(x)) \in A\}$ is everywhere non-meager. Conversely, the insights gained from this work have also led to variations on the Hoischen theorem that incorporate the ability to choose the approximating function so that the graphs of its derivatives cut a small section through a given null set or a given meager set.