Combinatorial Algebra meets Algebraic Combinatorics

Fifth Annual Meeting
Dalhousie University, Halifax, Canada

All talks will be in Room 319 of Chase Building.

Schedule:

	Friday January 18	**Saturday January 19**	**Sunday January 20**
9–10	Weyman	Bahturin	N. Bergeron
10–10:30	COFFEE BREAK	COFFEE BREAK	COFFEE BREAK
10:30	Stamate (10:30–11)	Iarrobino (10:30–11)	Zabrocki (10:30–11:30)
11:15–11:45	Emtander	Basili	
12–2	LUNCH BREAK	LUNCH BREAK	
2–3	F. Bergeron	Smith	
3–3:30	COFFEE BREAK	COFFEE BREAK	
3:30	Geramita (3:30–4:30)	Kosir (3:30–4)	

Abstracts:

1. Yuri Bahturin: "COMBINATORICS OF GRADED ALGEBRAS"

   We discuss new combinatorial methods, based on so called functional identities which were recently successfully applied to the description of gradings by groups on simple Lie and Jordan algebras. The combinatorial approach made it possible to relax the restrictions on the grading group, the dimension of the algebra and the of the field of coefficients usually imposed in this area.

2. Roberta Basili: "SOME REMARKS ON VARIETIES OF PAIRS OF COMMUTING UPPER TRIANGULAR MATRICES"

   It is known that the variety of pairs of $n \times n$ commuting upper triangular matrices isn't a complete intersection for infinitely many values of n; we show that there exists m such that this happens if and only if $n > m$. We also show that $m < 18$ and that it could be found by determining the dimension of the variety of pairs of commuting strictly upper triangular matrices.

3. Francois Bergeron: "SPACES OF POLYNOMIALS, BASKETBALLS, AND DESSINS D'ENFANTS"

4. Nantel Bergeron: "CHA, DDG, TA AND COINVARIANTS"

   With Li I have introduced a set of axioms which guarantee that the Grothendieck groups of a tower of algebras $\bigoplus_{n \geq 0} A_n$ can be endowed with the structure of graded dual Hopf algebras. Hivert and Nzeutzhap, and independently Lam and Shimozono constructed dual graded graphs from primitive elements in Hopf algebras. With Li and Lam, I apply the composition of these constructions to towers of algebras. We show that if a tower $\bigoplus_{n \geq 0} A_n$ gives rise to graded dual Hopf algebras then we must have $\dim(A_n) = r^n n!$ where $r = \dim(A_1)$.

   This shows that Combinatorial Hopf algebras obtain by this procedure falls into a very rigid framework and can potentially be classify. I will discuss the case $r = 1$ and show that the largest tower is related to the symmetric groups coinvariants.

5. Eric Emtander: "HYPERGRAPH ALGEBRAS"

   When studying the Stanley-Reisner ring of a simplicial complex, one is often interested in understanding the behaviour of the relations. This is the same thing as trying to understand a hypergraph. Recently I have studied some hypergraphs that are (in a combinatorial sense) natural generalizations of a couple of familiar classes of ordinary simple graphs. In this talk I will outline some of my results.

6. Tony Geramita: "POINTS IN P^n: THIN AND FAT (REFLECTIONS ON THE GEOMETRY OF THEIR HILBERT FUNCTIONS)"

   In this talk I will review some of the basic ideas concerning the Hilbert functions of points (thin and fat) in P^n - concentrating most of what I say in P^2. The situation for thin points is pretty well understood but that is not the case for fat points, even double points. That is a pity since their postulation is related to very interesting questions about higher secant varieties to the Veronese variety (among other things). I will discuss some recent work by Brian Harbourne, Juan Migliore and myself on some open questions relating to fat points (unfortunately, many still open!!!) and explain some combinatorial ideas we've used for the partial solutions we have found.

7. Anthony Iarrobino: "INVOLUTION ON THE NILPOTENT COMMUTATOR OF A NILPOTENT MATRIX"

   We consider the irreducible variety $\mathcal{N}_B$ of nilpotent elements of the commutator $\mathcal{C}_B$ of a nilpotent $n \times n$ Jordan matix B having Jordan blocks given by the partition P of n, over a field K. Fix a homomorphism: $\pi : C_B \to M_B$,

where M_B is a product of matrix algebras over K, with kernel the Jacobson radical $\mathfrak{J}_B$ of $\mathcal{C}_B$. The inverse image of the subvariety U corresponding to strictly upper triangular matrices, is a maximal nilpotent subalgebra $N_{B,sp}$ of C_B. R. Basili gave a specific homomorphism π, and parametrization of $N_{B,sp}$, that has been used by several. We describe an involution on $N_{B,sp}$, that is a generalized transpose. This involution underlies some of the symmetries we reported last year, in matrices ralated to the vanishing of elements of A^k, A generic in $N_{B,sp}$.

We pose several questions related to the open one of determining the Jordan partition of the generic element of $\mathcal{N}_B$. (Joint work with R. Basili)

8. Tomaz Kosir: "ON PAIRS OF COMMUTING MATRICES"

Let B be a nilpotent matrix and suppose that its Jordan canonical form is determined by a partition λ. Then it is known that its nilpotent commutator N_B is an irreducible variety and that there is a unique partition μ such that the intersection of the orbit of nilpotent matrices corresponding to μ with N_B is dense in N_B. We prove that the map D given by $D(\lambda) = \mu$ is an idempotent map. This answers a question of Basili and Iarrobino and gives a partial answer to a question of Panyushev. In the proof, we use the fact that for a generic matrix A in N_B the algebra generated by A and B is a Gorenstein algebra. Thus, a generic pair of commuting nilpotent matrices generates a Gorenstein algebra. This is a report on joint work with Polona Oblak.

9. Gregory G. Smith: "DETERMINANTAL EQUATIONS"

In this talk, we'll discuss sufficient conditions for the projective embedding of a variety by a complete linear series to be cut out by the 2*2 minors of a matrix of linear forms. We will give examples of such determinantal representations for toric varieties. Interpretations for the higher order minors of these matrices of linear forms will also be examined.

10. Dumitru Stamate: "KOSZUL RINGS VIA SEQUENTIALLY CM POSETS"

We prove that the associated graded ring of the incidence algebra of a finite poset is Koszul if and only if all open intervals are sequentially Cohen-Macaulay. This extends previous work of Woodcock and Polo. We also obtained a purely combinatorial characterization for when this associated ring is quadratic (e.g. when it is Koszul). Similar methods allow us to describe the Koszulness of affine semigroup rings extending results of Peeva, Reiner and Sturmfels. This is joint work with Vic Reiner, University of Minnesota.

11. Jerzy Weyman "THE CONJECTURES OF BOIJ AND SODERBERG ON BETTI DIAGRAMS OF FINITE FREE RESOLUTIONS"

In this talk I will describe the results of two recent papers: "*The existence of pure resolutions*" by Eisenbud, Floystad and myself (0709.1529) and "*Betti numbers of graded modules and cohomology of vector bundles*" by Eisenbud and Schreyer (0712.1843) which give a proof of some conjectures of Boij and Soderberg on Betti numbers of graded modules. In particular these results imply multiplicity conjectures of Herzog, Huneke and Srinivasan.

12. Mike Zabrocki "A FILTRATION OF CATALAN NUMBERS"

The q,t-Catalan numbers of Garsia and Haiman are a remarkable formula for the graded dimensions of the alternating space of diagonal harmonics. In joint work with Nantel Bergeron and François Descouens, we consider a filtration of the Catalan numbers based a level k by connecting the k-Schur functions of Lapointe-Lascoux-Morse to a definition and we look at combinatorial properties as well as ideas for possible filtrations of the space of diagonal harmonics. We also consider non-commutative analogues which have similar features to their commutative counterparts and, possibly because they lie in a space with more structure, are simpler to solve completely.