Formulas/Constants: Physics/Oceanography 4500/5500 A Atmospheric Physics

In addition to these formulas, there are many thermodynamic concepts and interrelationships between thermodynamic variables you must be familiar with, such as what types of quantities are conserved under what types of processes.

General:

Avogadro's number $N_A = 6.02 \times 10^{23}$ molecules/mole

1 hPa = 100 Pa

1 Pa = 1 N/m^2

dry adiabatic lapse rate: $\Gamma_d = 9.8$ °C/km

$k = 1.38 \times 10^{-23}$ J/K

$\gamma = c_p/c_v = 1.4$ (dry air)

Dry air:

$\mathrm{R}_d = 287.05$ J/(kg K) (mean gas constant for dry air)

$\mathrm{c}_{vd} = 717.5$ J/(kg K) (dry air)

$\mathrm{c}_{pd} = 1004.5$ J/(kg K) (dry air)

Mean molecular weight of dry air: $M_d = 28.94$ g/mole

Poisson relations:

$\mathrm{p}^{\frac{(1-\gamma)}{\gamma}}\mathrm{T} = \mathrm{const}$ or $T/p^{0.286}$ is constant (adiabatic process)

$pV^\gamma = \mathrm{const}$ (adiabatic process)

Ideal gas Law

The most basic form (for one type of molecule) is: $pV = NkT$, where N is the number of molecules, and the Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K.

An alternate form is:

$p = \rho RT$ (where R = k/m, m the mass per molecule)

For a mixture of gases, R is replaced by $< R >= R^*/ < M >$, where $< M >= \sum_j f_j M_j$. f_j is the number (or mole) fraction, and M_j is the molecular weight of each consituent. $R^* = 8.314$ J mol^{-1} K^{-1} is the universal gas constant.

$p = \rho R_d T_v$ (for moist air where T_v is the virtual temperature)

$\mathrm{R}_v = 461.5$ J/(kg K) (ideal gas constant for water vapor)

$\epsilon = M_v/M_d = 0.622$

$e = \rho_v R_v T$

latent heat of vaporization: $l_v = 2.5 \times 10^6$ J/kg

latent heat of fusion: $l_f = 3.34 \times 10^5$ J/kg

approximate density of water $\rho_l = 1000$ kg/m^3

Thermodynamic Definitions:

$H = U + PV$

$Q = dH/dt$ (constant pressure process)

$Q = dU/dt$ (constant volume process)

$dU/dt = Q + W$ (first law of thermodynamics)

This can also be written: $\Delta U = \Delta Q + \Delta W$, for differences in finite time processes, rather than rates.

$W = -pdV/dt$

$Q = C_p \frac{dT}{dt}$ (constant pressure process)

$Q = C_v \frac{dT}{dt}$ (constant volume process)

specific heat $c_p = C_p/M$, where M is the mass

ideal gas: $U = C_v T$ and $H = C_p T$

$G = H - TS$

$dse = c_p T + gz$ (Definition of dry static energy)

$\frac{\partial G}{\partial p} = V$

$\frac{\partial G}{\partial T} = -S$

Hydrostatic Relationship: $dp/dz = -\rho g$

Thickness Relationship: $z_2 - z_1 = \frac{R}{g} < T > ln(\frac{p_1}{p_2})$

Water vapor

Mean molecular weight of water vapor: $M_v = 18.015$ g/mole

Mean molecular weight of air: $M = f_d M_d + f_v M_v$, where f_d and f_v are the molar fractions of dry air and the vapor ($f_d + f_v = 1$).

Accurate expression for dependence of saturation vapor pressure on temperature:

$$ln\frac{e_s}{e_{so}} = 6808(\frac{1}{T_0} - \frac{1}{T}) - 5.09 ln\frac{T}{T_0}$$

where, $T_0 = 273.15$ K, $e_{s0} = 6.11$ hPa, T in Kelvin

Approximate expression for dependence of saturation vapor pressure (in hPa) on temperature over water (T in Kelvin, $e_{so} = 6.11$ hPa):

$$e_s = e_{so} exp[19.83 - (5417/T)]$$

Expression for dependence of saturation vapor pressure (in hPa) on temperature over ice (T in Kelvin, $e_{so} = 6.11$ hPa):

$$e_{si} = e_{so} exp[22.49 - (6142/T)]$$

Clausius-Clapeyron Equation:

$$\frac{1}{e_s}\frac{de_s}{dT} = \frac{l_v}{R_v T^2}$$

dew point: $e = e_s(T_d)$

relative humidity: $r = e/e_s$

specific humidity: $q = \rho_v/\rho = w/(1 + w)$

mass mixing ratio: $w = \rho_v/\rho_d = \epsilon e/p_d \approx \epsilon e/p$

$\epsilon = R_d/R_v = 0.622$

$e = \frac{w}{\epsilon + w} p$

$c_{v,w} = 1390$ J/(kg K) (specific heat at constant volume of water vapor, 273 K)

$c_{p,w} = 1850$ J/(kg K) (specific heat at constant pressure of water vapor, 273 K)

You can assume that all specific heats are independent of temperature.

Specific heat of a mixture: $c_p = \sum_j c_p^j w_j$, where w_j is the mass fraction ($w_j = M_j/M$), and c_p^j is the specific heat of each component.

From this you can show, that for a moist air parcel $c_p = (1 + 0.87q)c_{pd}$.

Approximate expression for z_{LCL}:

$$z_{LCL} - z_o = \frac{T_o - T_{do}}{8}$$

,

(in km), where T_o is the temperature at the ground and T_{do} is the dew point temperature at the ground.

Approximate form for the temperature at the lifting condensation level:

$$T_{LCL} = \frac{9.8T_{do} - 1.8T_o}{8}$$

Entropy

$$S = S_0 + C_V ln(\frac{TV^{\gamma-1}}{T_0 V_0^{\gamma-1}})$$

where T_0 and V_0 are some reference pressure and temperature (often chosen 273.15 K and 1000 hPa).

$$S - S_0 = C_p ln(T/T_0) - Nkln(p/p_0)$$

Potential Temperature:

$\theta = T(p_0/p)^{\frac{(\gamma-1)}{\gamma}} = T(p_0/p)^{0.286}$, where $p_0 = 10^5$ Pa = 1000 hPa

Virtual Potential Temperature:

$\theta_v = T_v(p_0/p)^{0.286}$, where $p_0 = 10^5$ Pa

Isobaric Mixing:

$$T = \frac{C_{p1}}{C_{p1} + C_{p2}} T_1 + \frac{C_{p2}}{C_{p1} + C_{p2}} T_2$$

$$e = \frac{N_{w1} + N_{a1}}{N_{w1} + N_{w2} + N_{a1} + N_{a2}} e_1 + \frac{N_{w2} + N_{a2}}{N_{w1} + N_{w2} + N_{a1} + N_{a2}} e_2$$

where the N's are number of moles, or number of molecules.

$de_s/dp = \rho_v/\rho_w$ (effect of air on e_s, ρ_v the density of water vapor, and ρ_w the density of water.)

Raoult's Law: $e_s(f_{H_2O}, T) = f_{H_2O}\, e_s(T) = e_s(T)\frac{N_{H20}}{N_{H20} + N_X}$, where N_X is the number of moles of X, and f_{H_2O} is the molar fraction of water in the liquid.

Henry's Law: $c = \frac{p}{K(T)}$, where $K = \frac{\beta\sqrt{2\pi mkT}}{\alpha}$, c is the concentration of the gas in the solvent, p the pressure of the gas in the air, β is a constant, and α is the mass accommodation coefficient. The units of c are usually M = moles/litre, so units of K would be (hPa × litre)/moles.

Condition for equilibrium between a vapor and droplet of size r: $g_v(p,T) = g_w(p,T) + \frac{2\sigma}{r\rho_w}$, where g_v is the Gibbs free energy of the vapor, g_w is the Gibbs free energy of the bulk liquid phase, σ is the surface free energy per unit area (surface tension), and ρ_w is the density of the liquid.

Kelvin equation: $\frac{e_{sr}}{e_{s\infty}} = exp(\frac{2\sigma}{r\rho_w R_v T})$, where r is the radius of the drop.

Approximate form of Kelvin equation: $\frac{e_{sr}}{e_{s\infty}} = exp(\frac{0.00108}{r})$, where r is the radius of a drop in microns.

Precipitable water: $d_w = \frac{1}{\rho_w}\int_0^\infty q\rho dz$

$\Gamma_{dew} = \frac{g}{\epsilon l_v}\frac{{T_d}^2}{T}$

Virtual Temperature: $T_v = T(1+0.608q) = \frac{M_d}{M}T = \frac{R}{R_d}T$

An important use of virtual temperature is the ideal gas law for a moist air parcel ($p = e + p_d$ total pressure, and $\rho = \rho_v + \rho_d$ total density): $p = \rho R_d T_v$, where p is the total pressure of the air parcel, and ρ the total density.

Moist adiabatic lapse rate: $dT/dz = -\frac{g}{c_p} - \frac{1}{c_p}\frac{d}{dz}(w_s l_v)$, where $c_p = \frac{Mc_w + M_d c_{pd}}{M_d}$, where $M = M_w + M_v$. (This expression for specific heat is not a general expression for c_p of a moist air parcel, but only a form useful as a parameter in this equation.)

Approximate expression for MALR: $\frac{dT}{dz} = -\frac{g}{c_p}\frac{1+l_v w_s/RT}{1+{l_v}^2 w_s/c_p R_v T^2}$

moist static energy (per unit dry mass): $mse = (c_{pd} + w_t c_w)T + l_v w + gz$

$\theta_d = T(p_0/p_d)^{R_d/c_p}$, where $p_0 = 10^5$ Pa and $c_p = \frac{Mc_w + M_d c_{pd}}{M_d}$.

$T_e = T + \frac{l_v w}{c_{pd} + wc_w}$

psychometric equation for wet bulb temperature: $e = e_s(T_w) - \frac{pc_{pd}}{\epsilon l_v}(T - T_w)$

$\theta_e = \theta_d exp(\frac{l_v w_s}{c_p T})$, equivalent potential temperature (for saturated air)

$\theta_e = \theta_d(LCL)exp(\frac{l_v(T_{LCL})w}{c_p T_{LCL}})$, equivalent potential temperature (for unsaturated air)

approximate form for θ_e: $\theta_e = \theta exp(\frac{2675w}{T_{LCL}})$.

approximate form for θ_e: $\theta_e = \theta exp(\frac{l_v w}{c_p T_{LCL}})$.

$$CAPE = -R_d \int_{P_{LFC}}^{P_{LNB}} (T_{vp} - T_{vs}) d\, lnp$$

$$V_{zf} = \sqrt{2\, CAPE}$$

$$CINE = R_d \int_{P_i}^{P_{LFC}} (T_{vp} - T_{vs}) d\ lnp$$

Buoyancy:

$$\mathbf{F} = -g \int_V (\rho' - \rho) dV$$

where g is the gravitational acceleration, ρ' is the density of the parcel, and ρ is the density of the background atmosphere.

An expression for the buoyancy acceleration is:

$$B = g\frac{(\rho - \rho')}{\rho}$$

or, in terms of virtual temperature (if effects on density due to varying amounts of water vapor are neglected, can use temperature alone):

$$B = g\frac{(T_v' - T_v)}{T_v}$$