**Formulas: PHYS/OCEA 4520/5520** *Introduction to Atmospheric Science*

*Constants:*

Radius of the earth $R_e = 6.37 \times 10^6 m$

$N_A = 6.02 \times 10^{23}$ molecules/mole

$g_0 = 9.80$ m/s$^2$

1 Pa = 1 N/m$^2$

$p_0 = 10^5$ Pa

$\sigma = 5.67 \times 10^{-8} J s^{-1} m^{-2} K^{-4}$

$\Gamma_d = c_p/g = 9.8$ °C/km

$k = 1.38 \times 10^{-23}$ J/K

$\gamma = c_p/c_v = 1.4$ (dry air)

$R_d = 287.05$ J/(kg K) (mean gas constant for dry air)

$c_{vd} = 717.5$ J/(kg K) (dry air)

$c_{pd} = 1004.5$ J/(kg K) (dry air)

$c_{pw} = 1004.5$ J/(kg K) (water vapor)

$M_d = 28.94$ g/mole

$M_w = 18.015$ g/mole (molecular weight of water)

$R_d = 287.0$ J/(kg K)

$R_v = 461.5$ J/(kg K)

$R^* = 8.3145$ J mol$^{-1}$ K$^{-1}$

$\epsilon = M_v/M_d = 0.622$

$L_v = 2.5 \times 10^6$ J/kg

$L_m = 3.34 \times 10^5$ J/kg

$\rho_{water} = 1000$ kg/m$^3$

$\rho_{ice} = 917$ kg/m$^3$

$\sigma = 5.67 \times 10^{-8} J s^{-1} m^{-2} K^{-4}$

*Ideal gases:*

$p = \rho R T$

$R = 1000 R^* / < M >= 1000 R^*/(f_d M_d + f_v M_v)$

$< M >= f_d M_d + f_v M_v$

$f_d$ : molar fraction of dry air $(= p_d/p)$

$f_v$ : molar fraction of water vapor $(= e/p)$

$p = \rho R_d T_v$

$p_d = \rho_d R_d T$

$e = \rho_v R_v T$

$p = e + p_d$

$\rho = \rho_v + \rho_d$ (in a cloud would include liquid water density $\rho_l$)

$T_v = T(1 + 0.608\, w) = (M_d / < M >) T = (R/R_d) T$

$du = c_v dT$

$c_v dT = dq - p d\alpha$

$c_p = c_v + R$

$h = c_p T$

*Thermodynamic Definitions:*

$\alpha = 1/\rho$ (specific volume)

$dU = dQ - dW$ (first law)

$dH = dQ + V dp$ (first law)

$dq = du + p d\alpha$ (first law)

$dh = dq + \alpha dp$ (first law)

$dW = pdV = \int_{V_1}^{V_2} = pdV$

$du = dU/M$, $dh = dH/M$, $dw = dW/M = pdV/M = pd\alpha$

$H = U + PV$

$h = u + p\alpha$

$c_p = (dq/dT)_p$

$c_v = (dq/dT)_v$

$c_p = C_p/M$

$dp/dz = -\rho g$ (Hydrostatic relationship)

$Z = \Phi(z)/g_0$

$dZ = (g/g_0)dz$

$d\Phi = gdz$

$\Phi_2 - \Phi_1 = -R_d \int_{p_2}^{p_1} T_v dp/p$

$Z_2 - Z_1 = (R_d/g_0) \int_{p_2}^{p_1} T_v dp/p = (R_d \overline{T}_v/g_0)\, ln(p_1/p_2)$

$e_s = e_{so} exp[22.49 - (6142/T)]$ (T in Kelvin, $e_{so} = 6.11$ hPa):

C-C equation: $(1/e_s)(de_s/dT) = (L_v/R_v T^2)$

$e = e_s(T_d)$

relative humidity: $RH = e/e_s \times 100\%$

specific humidity: $q = m_v/(m_d + m_v) = \rho_v/\rho = w/(1+w)$

mass mixing ratio: $w = \rho_v/\rho_d = \epsilon e/p_d = \epsilon e/(p - e)$

$e/p = w/(\epsilon + w)$

$w_s = 0.622 e_s(T)/p$

$\theta = T(p_0/p)^{0.286}$, $p_0 = 1000$ hPa

$\theta_e \approx \theta exp(L_v w/c_p T_{LCL})$

moist static energy: $MSE = c_p T + gz + L_v q$

dry static energy: $DSE = c_p T + gz$

$B = g(\rho - \rho')/\rho'$

$\rho'$: parcel density

$\rho$: background density

$B = g(T_v' - T_v)/T_v$ (ignoring possible condensate)

$B \approx g(T' - T)/T$

$d^2 z/dt^2 + N^2 z' = 0$

$N^2 = (g/T)(\Gamma_d - \Gamma)$

$\eta = (Q_1 - Q_2)/Q_1 = 1 - (T_2/T_1)$

$\Delta Q = c_p M \Delta T$

*Dynamics:*

Pressure Gradient Acceleration in One Dimension:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{dp}{dx}$$

*Radiation*

$d\Omega = dA/R^2$

$I(P, \mathbf{n}, \lambda) = dE/(d\Omega\, dA\, d\lambda\, dt)$

$A(P, \lambda) = \int_0^{2\pi} d\phi \int_{-\pi/2}^{\pi/2} I(\lambda, \theta, \phi)\, sin\theta\, d\theta$

$F(P, \lambda) = \int_{2\pi} I(\lambda)\, cos\theta\, d\omega = \int_0^{2\pi} d\phi \int_0^{\pi/2} I(\lambda, \theta, \phi)\, cos\theta\, sin\theta d\,\theta$

$F = \sigma T^4$ (black body)

$F = \epsilon\sigma T^4$ (grey body)

$dI_\lambda/dz = -I_\lambda \rho r k_\lambda = -I_\lambda K_\lambda N\sigma$

$r$: mass mixing ratio of absorber

$K_\lambda$: Scattering efficiency

$\sigma$: areal (geometric) cross-section

$k_\lambda$: mass absorption cross section

$N$: molecular number density

$\rho$: mass density

Transmission : $T = e^{-\tau/cos\theta}$

$I_\lambda(z, \theta) = I_\infty e^{-\int_z^\infty k_\lambda r\rho dz/cos\theta}$

$\tau_\lambda = -\int_z^\infty k_\lambda r\rho dz$

need def in terms of scattering efficiency

$I_\lambda(z, \theta) = I_\infty e^{-\tau_\lambda/cos\theta} = T_\lambda I_\infty$

$dT/dt = -1/(\rho c_p)\ \times dF/dz$ (F is the net SW + LW radiative flux)

molecular scattering cross-section $k_\lambda \propto 1/\lambda^4$ (small x "Rayleigh regime")

size parameter $x = 2\pi r/\lambda$

Scattering phase function $P(\theta, \phi; \theta', \phi')$

$I_{\nu,\infty} = B_\nu(T_s)e^{-\tau_\nu} + \int_0^\infty B_\nu[T(z)]e^{-\tau_\nu(z)} k_\nu \rho r dz$

weighting function: $w_\nu = e^{-\tau_\nu(z)} k_\nu \rho r$