Chapter 1: Introduction (T1)

1. Specify two differences between the stratosphere and troposphere.

2. In Northern Hemisphere winter, do surface lows tend to form over ocean or land? Explain.

3. We discussed in class two climates referred to as the "Mediterranean" (think California or Rome, roughly 35 ^{o} N) and "mid-latitude continental" (think Winnipeg or Moscow, or roughly 50 - 55 ^{o} N).
(i) Plot the seasonal variation in precipitation of these two climates from January to December. Assume Northern Hemisphere. Label the two curves.
(ii) What is the main factor influencing the overall amount of winter precipitation in mid-latitude continental climates?
(iii) What is the main factor influencing the overall amount of winter precipitation in Mediterranean climates?
(iv) What is the main factor influencing the overall amount of summer precipitation in Mediterranean climates?
(v) Plot the seasonal variation in temperature for these two climate types from January to December. Again assume Northern Hemisphere. Pay close attention to the relative amplitude of the seasonal cycles at the two locations, and the relative values of the temperatures at the two locations over the course of a year.
(iv) What is the main factor influencing the amplitude of the seasonal cycle in temperature for continental climates?

4. (i) Assume the stratosphere lies between the 150 hPa and 1 hPa pressure levels. What is the mass of the entire stratosphere? ?
(ii) Air enters the stratosphere in the tropics, and exits the stratosphere in mid-latitudes. Air enters the stratosphere at a rate of 2.4×10^{17} kg/year. Estimate the average residence time of an air parcel in the stratosphere.

5. What are two differences between the troposphere and stratosphere?

6. The prevailing wind direction in mid-latitudes is westerly (i.e. from the west). Explain, with reference to the large scale temperature structure of the troposphere. Please use a force diagram to motivate your reasoning.

7. Plot how the mean rainfall rate depends on sea surface temperature (SST) in the tropics. Give specific SST values if possible.

8. Most of the water in the deep ocean is believed to have originated from the surface waters of the North Atlantic and the Antarctic. What is the observational evidence in support of this theory?

9. (i) Plot the seasonal variation of rainfall and temperature (January through December) at a location in the Northern Hemisphere with a "Mediterranean" climate.
(ii) Discuss the reasons for the seasonal rainfall variation shown in (i).

10. Errors in weather forecasts are divided into two main categories. Specify the two main sources of error in weather forecast models, and give a specific example within each error category.

11. The highest rainfall rates in North America tend to occur along the West coast. Explain.

12. (i) Subtropical oceanic highs influence the spatial distribution of rainfall. Explain. A diagram may be helpful.

(ii) Specify a location where the rainfall is influenced by one of these highs. How is the rainfall at this location influenced by the high?

13. (i) What are two physically distinct ways of driving a circulation?
(ii) Give an example of each type of circulation, either in the atmosphere, the ocean, the solid earth, or in everyday life.

14. (i) Suppose it is summer. Would you expect surface highs to tend to form over the ocean or the land? Explain.
(ii) Suppose it is winter. Would you expect surface highs to tend to form over the ocean or the land? Explain.

15. Let the vertical axis be height, and the horizontal axis be latitude. Assume the surface pressure is 1000 hPa. Make a rough plot of the typical variation of the height of the 500 hPa surface against latitude from the equator to the North Pole. Show plots for both Northern Hemisphere summer and winter. You can assume that the height of the 500 hPa surface has no seasonal variation at the equator.

16. The existence of the stratospheric Brewer-Dobson circulation was originally inferred from water vapor measurements taken near 12 km above England. Explain.

17. The density of the water in the Black Sea is reduced by the input of fresh water from the surrounding rivers. It is therefore less dense than the warm but salty water of the Mediterranean. These two water bodies are connected through the Bosporus Strait. How would you expect the surface current in this Strait to flow? From the Mediterranean to the Black Sea, or vice versa? Explain.

18. What are two differences between synoptic rain/convective rain?

Chapter 2: The Earth System (T2)

1. Specify and briefly discuss one long term geological mechanism whereby CO_2 ends up being removed from the atmosphere.

2. The exposure of calcium silicate rocks to the atmosphere, and their subsequent weathering, can reduce the level of CO_2 in the atmosphere. Explain how this occurs. A simple diagram may be helpful. You do need to show the exact chemical reactions.

3. What is the main geological source of CO_2 to the atmosphere?

4. Specify and briefly discuss two carbon cycles within the earth system. These brief discussions may discuss the various forms of carbon in the cycle, the lifetimes of the carbon reservoirs, and one or two reactions involving carbon in the cycle, if any. Please be as specific as possible.

Chapter 3: Atmospheric Thermodynamics (T3)

1. Use the first law of thermodynamics to show that dry static energy is conserved for vertical displacements of air parcels which are adiabatic and hydrostatic. Assume the air parcel is an ideal gas. Start from the most appropriate form of the first law of thermodynamics.

2. Circle which of the following properties of an air parcel are conserved under dry, adiabatic, hydrostatic ascent with no mixing:

e	T_d	w	$dse = cpT + gz$	f_v	T_v
e/e_s	θ	e_s	u	h	q

Answer: (1) w, PT, fv, q, dse conserved.

3. Knowledge of the dew point temperature during the day is sometimes helpful in predicting how cold temperatures at the surface may get overnight. Explain.
Answer: It is hard for overnight surface temperatures to go below daytime Td since condensation occurs when T reaches Td. This slows down T decrease due to radiative cooling.

4. How can a knowledge of the the temperature, water vapor mixing ratio, and T_{LCL} of air parcels near the surface be used to help estimate the top outflow level of convective clouds in the tropics? You may use a diagram if it is helpful
Answer: Can use to calculate equivalent potential temperature, and top outflow altitude near height at which EQPT of parcel coming up from the surface equals PT of background atmosphere.

5. For gases, which is larger: c_p or c_v? Explain in words, and using the first law of thermodynamics, or other thermodynamic relationships.

6. Precipitation heats the atmosphere by condensational heating. The globally averaged precipitation rate is 100 cm/year.
(i) Convert this rain rate to an atmospheric heating rate in units of W/m^2. (1 W = 1 J/s.)
(ii) Assume that this condensational heating is distributed uniformally in the troposphere (1000 hPa - 200 hPa). Calculate the mass per unit area of the troposphere (in kg/m^2).
(iii) Calculate the average rate of temperature change of the troposphere due to condensational heating (K/day). You can can assume that the specific heat of the atmosphere equals the specific heat of dry air.
(iv) How does the troposphere balance condensational heating to keep temperatures roughly constant?

7. Specify two factors that affect the buoyancy acceleration B of an air parcel rising inside a cloud.

8. Water vapor molecules which condense on to liquid water tend to increase the temperature of the water. Why?

9. What are two process which can affect the potential temperature of an air parcel in the atmosphere?

10. For ideal gases, the specific heat at constant pressure c_p is larger than the specific heat at constant volume c_v. Explain, with reference to the first law of thermodynamics.

11. A container is filled with two gases. Gas A has a pressure of 20 hPa and a molecular weight of 20 g/mole. Gas B has a pressure of 40 hPa and a molecular weight of 35 g/mole.
(i) What is the mean molecular weight of the mixture?
(ii) What is the gas constant R of the mixture?

12. The surface pressure is 1000 hPa. Approximately what fraction of the total atmospheric mass is located between the 600 hPa and 400 hPa pressure surfaces?

13. A heating of $dQ = 5000J$ is applied to a 2 kg parcel of dry air under isobaric (constant pressure) conditions. You can assume that dry air is an ideal gas.
(i) What is the change in enthalpy H of the air parcel?
(ii) What is the change in temperature of the air parcel?
(iii) What is the change in internal energy U of the air parcel?
(iv) What is the work done by the air parcel?

14. There is a mixed phase cloud at a temperature below 0 °C in which both ice crystals and supercooled water droplets are present. The water vapor pressure e lies between the saturated water vapor pressures of water (e_s) and ice (e_{si}).
(i) Compare the magnitudes of the evaporative (E) and condensational (C) fluxes for the supercooled liquid water droplets. Would you expect the water droplets to be growing or shrinking?
(ii) Compare the magnitudes of the evaporative (E) and condensational (C) fluxes for the ice crystals. Would you expect the ice crystals to be growing or shrinking?

Chapter 3: Water Variables

1. An air parcel at 300 K and a total pressure of 1000 hPa has a relative humidity of 40 %.
(i) What is the water vapor pressure e of the air parcel?
(i) What is the dew point temperature of the air parcel?
(ii) What is the mean molecular weight of the air parcel?
(iii) What is the virtual temperature T_v of the air parcel?
(iv) What would be the relative humidity of the air parcel if it is lifted adiabatically to 900 hPa?

2. An air parcel at a pressure $p = 900$ hPa and temperature 20 °C has a water vapor mass mixing ratio $w = 0.01$ kg H_2O/kg dry air.
(i) What is the dew point temperature T_d of the air parcel?
(ii) Suppose the air parcel described above is originally at an altitude of 1 km on the upstream side of a mountain range, and then flows over the mountain range. In the process, its mass mixing ratio w is reduced to one quarter of its original value. What is the final temperature of the air parcel when it returns to an altitude of 1 km on the other side of the mountain range? (Hint: you can assume that moist static energy (MSE) is conserved throughout this process.)
(iii) What is the final relative humidity of the air parcel at 1 km?

3. An air parcel is lifted up adiabatically from the surface ($p = 1000$ hPa), reaches its lifting condensation level at 800 hPa, and is then lifted moist adiabatically to 100 hPa where the temperature is very cold (say 200 K). Make a very rough schematic plot of how the lapse rate dT/dz would vary with vertical axis should be pressure. Label special values of pressure or lapse rate if there are any.
Answer: Constant value of dT/dz at dry value up to the LCL, sudden decrease in dT/dz at LCL, and slow convergence to dry value in upper troposphere.

4. 10 liters of air at 25 °C with a relative humidity of 60 % are compressed isothermally to 2 liters. The saturation vapor pressure at 25 °C is approximately $e_s = 30$ hPa.
(i) What is the original vapor pressure e of the air parcel?
(ii) What is the original mass of water vapor in the air parcel?
(iii) Calculate the mass of water vapor that must have condensed if the relative humidity of the air parcel does not exceed 100 %?

5. An air parcel at 280 K and a total pressure of $p = 800$ hPa has a relative humidity of 40 %. (Solve questions mathematically, rather than using a skewT-lnp diagram)
(i) What is the water vapor pressure e of the parcel?
(ii) What is the water vapor density ρ_v of the parcel?
(iii) What is the dew point temperature of the air parcel?
(iv) What would be the relative humidity of the air parcel if it is transported dry adiabatically to 1000 hPa?
Answer: Invoke conservation of PT in (iv) to find temperature at the surface, to get es.

1. An air parcel at 1000 hPa has a temperature of 30 ^{o}C and a water vapor mass mixing ratio $w = 12$ g/kg. Using the skewT - lnp chart provided:
(i) Estimate the p_{LCL} of the air parcel. (LCL refers to Lifting Condensation Level)
(ii) Estimate the equivalent potential temperature θ_e of the air parcel.
(iii) Plot the lapse rate dT/dz of the air parcel against pressure, assuming that it is lifted upward adiabatically from the starting pressure $p = 1000$ hPa to the LCL, and then moist adiabatically to a final pressure $p = 200$ hPa. I am looking for an approximate "schematic" plot only, but please give numbers for pressure or dT/dz where you can.

2. An air parcel at 1000 hPa has a temperature $T = 25$ ^{o}C and a dew point temperature $T_d = 7$ ^{o}C. In (i) - (iv) below, make sure to show all reasoning on the skew T - ln p diagram provided.
(i) Using the skew T - ln p plot, estimate the water vapor mass mixing w of the air parcel.
(ii) What is is the p_{LCL} of the air parcel?
(iii) The air parcel is transported over a mountain whose peak is at a pressure $p = 400$ hPa. During the forced ascent, all condensate is immediately rained out. Using the skew T - ln p plot, estimate the w of the air parcel at the top of the mountain.
(iv) Suppose the air parcel is advected dry adiabatically down to 1000 hPa on the downwind side of the mountain. What is the temperature of the air parcel when it reaches 1000 hPa?
(iv) RH when descends to 1000 hPa?
(v) Rather than using a skew T - ln p diagram, calculate the relative humidity of an air parcel with $T = 25$ ^{o}C, dew point temperature $T_d = 7$ ^{o}C, and pressure $p = 1000$ hPa.

3. An air parcel at $p = 1000$ hPa has a temperature $T = 30$ ^{o}C, and a dew point temperature $T_d = 3$ ^{o}C. Use the skew T - ln p diagram provided to answer the following questions.
(i) What is the water vapor mass mixing ratio w of the parcel?
(ii) The air parcel is lifted dry adiabatically to its lifting condensation level. What is p_{LCL}?
(iii) What is the dew point temperature T_d of the air parcel at the LCL?
(iv) What is the equivalent potential temperature θ_e of the parcel?
(v) The air parcel is then lifted moist adiabatically to $p = 300$ hPa. What is the water vapor mass mixing ratio w at $p = 300$ hPa?
(vi) What is the temperature of the parcel at $p = 300$ hPa?
(vii) The parcel is then moved dry adiabatically to the surface pressure $p = 1000$ hPa. What is the temperature of the parcel at the surface?
(viii) What is the water vapor mass mixing ratio w at the surface?
(ix) What is the dew point temperature at the surface?
(x) What is the change in potential temperature of the parcel during this process? (from starting at the surface and then ending up back at the surface again)

4. An air parcel at $p = 1000$ hPa has a temperature $T = 20$ ^{o}C, and a dew point temperature $T_d = 1$ ^{o}C. Use the skew T - ln p diagram to answer the following questions.
(i) What is the water vapor mass mixing ratio w of the parcel?
(ii) What is the equivalent potential temperature θ_e of the parcel?
(iii) The air parcel is lifted dry adiabatically to its lifting condensation level. What is p_{LCL}?
(iv) What is the dew point temperature T_d of the air parcel at the LCL?
(v) The air parcel is then lifted moist adiabatically to $p = 500$ hPa. What is the water vapor mass mixing ratio w at $p = 500$ hPa?
(vi) What is the temperature of the parcel at $p = 500$ hPa?

(vii) The parcel is then moved dry adiabatically to the surface pressure $p = 1000$ hPa. What is the temperature of the parcel at the surface?
(viii) What is water vapor mass mixing ratio w at the surface?
Answer: w is constant during descent in (viii) and equal to saturated value at 500 hPa from (v).

Chapter 4: Radiative Transfer (T4)

Chapter 4: General

1. The high altitude clouds generated by exhaust from aircraft flying near the tropopause (contrails) will affect the upward SW and LW radiative flux at the top of the atmosphere. Would you expect the net radiative effects of such clouds to exert a net warming or net cooling on the earth? Explain.

2. In a single figure, plot the annual mean SW energy absorbed by the earth (atmosphere + surface) as a function of latitude, and the annual mean LW energy emitted by the earth (atmosphere + surface) as a function of latitude. Allow the latitude to vary from 90 °S to 90 °N.

3. On December 21, the location on earth with the highest incident solar flux at the top of the atmosphere is the South Pole. However, surface temperatures at the South pole remain quite cold. Discuss two reasons why the large TOA 24 hour solar flux does not give rise to high surface temperatures.
Answer: Could say, low solar zenith angles cause more molecular SW scattering, solar reflection from snow and ice, large heat of melting making it difficult to melt ice and get surface temperatures above zero, or high altitudes.

4. The upward, TOA (Top of the Atmosphere) LW flux tends to be large over tropical deserts. Give two reasons for this.

5. The effective emitting temperature of a planet can be calculated from the value S_0 of the solar flux incident on the planet, and its solar reflectivity A. For some planets, however, the observed T_E is larger than what would be calculated from S_0 and A. Give one reason why this might be the case.

6. (i) An air parcel in the atmosphere is said to be in "radiative equilibrium". What is meant by this statement?
Answer: Radiative Heating = Radiative Cooling. Small contribution to Heat Equation from Horizontal + Vertical Temperature Advection. e.g. subsidence warming or expansion cooling plays a secondary role in the heat budget.
(ii) Are air parcels in the troposphere in radiative equilibrium? Why or why not?
Answer: The troposphere is not in radiative equilibrium, mainly because subsidence warming plays an important role in the heat budget. This downward motion is forced by ascent and condensational warming inside clouds.

7. Suppose that all the clouds on our planet contained much more soot, and therefore absorbed much more solar radiation than they do now. Speculate on how this might affect average rainfall rates at the surface. Credit given for any reasonable argument. Assume that the cloud distribution does not change.
Answer: In the current atmosphere, the radiative equilibrium temperature profile has a lapse rate that exceeds the dry and moist adiabatic lapse rates. This causes dry and moist convection. The heat released within clouds tends to drive the tropical temperature profile to a moist adiabatic profile. Any additional source of heat in the atmosphere would stabilize the atmosphere and reduce the need

for condensational heating to stabilize it. Atmospheric solar absorption from soot should therefore reduce rainfall rates.

Chapter 4: Greenhouse Effect

1. Suppose that, at a particular wavelength λ, the emission to space is dominated by atmospheric emission from a well mixed greenhouse gas X. Suppose also that the concentration of this gas in the atmosphere were doubled.
(i) How would this doubling affect the average height of emission to space from gas X at this wavelength? Show schematic plots of the density ρ_X and transmission probability $T_\lambda(z)$ of X, before and after it is doubled. Let the vertical axis refer to altitude. Here $T_\lambda(z)$ refers to the probability that a photon of wavelength λ emitted by X at a height z will escape to space. Show how the changes in ρ_X and $T_\lambda(z)$ affect the average height of emission to space. Indicate on the plots the curves which refer to before/after doubling X.
(ii) Suppose that most of the emission to space from gas X is coming from the troposphere. How would you expect the change in average height of emission shown in (i) to affect the magnitude of the LW emission to space by X at this wavelength? Explain.

2. Specify and briefly discuss two factors which influence the radiative forcing of a greenhouse gas (i.e. the amount by which a gas reduces the outgoing LW radiation to space.)

3. (i) Do high clouds typically warm or cool the earth's climate? Explain.
(ii) Do low clouds typically warm or cool the earth's climate? Explain.

4. Define the effective emitting temperature T_E of a planet like our earth.

5. A planet has an isothermal atmosphere. Would you expect the planet to have a greenhouse effect? Why or why not?

Chapter 4: Scattering And Absorption Coefficient

1. (i) Plot the scattering efficiency K_λ of a spherical, non absorbing, particle against its size parameter x. Let the vertical axis refer to the scattering efficiency, and allow it to vary on a linear scale from 0 to 5. Let the horizontal axis refer to the size parameter, and allow it to vary on a logarithmic scale from 0.1 to 100. Indicate the "Rayleigh regime" on your plot.
(ii) Suppose that a cloud were to consist of a collection of identical water droplets, all having a same radius similar to that of visible light. Would you expect such a cloud to have a color? Explain.

2. On a sunny day, the solar radiation is mainly composed of the direct beam from the sun, plus a component due to molecular scattering in the atmosphere. Which component is more isotropic? Explain.

3. The sun is directly overhead (i.e. $\theta = 0$). The mass absorption coefficient of ozone at 300 nm is $k = 430$ m^2/kg. The transmittance of the atmosphere is $T = 0.02$. Assume that the only process in the atmosphere affecting light at 300 nm is ozone absorption.
(i) What is the optical depth of the atmosphere at 300 nm?
(ii) What is the ozone column mass per unit area (kg/m^2)?
(iii) This calculation ignores molecular scattering. Would inclusion of this process increase or decrease your estimate of the ozone column? Explain.

4. The scattering properties of an object can be characterized by a function called the scattering phase function $P(\theta, \phi; \theta', \phi')$.
(i) Explain what the scattering phase function is, in words.
(ii) How does the scattering phase function typically change as the size parameter of an object is changed from $x \ll 1$ to $x \gg 1$?
(iii) What is an example of scattering with $x \ll 1$?
(iv) What is an example of scattering with $x \gg 1$?

5. Molecules in the atmosphere scatter LW (thermal) radiation. However, this process has virtually no effect on the climate of the earth, and is usually ignored. Explain.
Answer: The size parameter x for LW radiation incident on molecules is extremely small. The wavelength of LW radiation is about 10 microns, or 10,000 nm, and the size of a molecule is less than a nm. Therefore one is in the Rayleigh regime where the scattering efficiency scales as 1/lamda to the fourth power, or x to the fourth power. Since x is extremely small, the scattering efficiency is extremely small. Another way of imagining it is if one extrapolated the figure above showing the scattering optical depth of the atmosphere in the visible spectrum to the LW spectrum. The optical depth would progressively decrease as lambda increased, and be vanishingly small in the LW range (i.e. way past red light). Of course the optical depth of the atmosphere of LW radiation with respect to emission and absorption by greenhouse gases is not small. This is a fundamentally process than molecular scattering.

6. Allow the x axis to vary between 300 nm (UV light) and 750 nm (red). Allow the y axis to vary from an optical depth of 0.01 to 10 on a logarithmic scale. On the same graph, make rough schematic plots of the following quantities as a function of wavelength.
(i) The optical depth of the atmosphere due to molecular scattering (overhead sun).
(ii) The optical depth of the atmosphere due to ozone absorption
(iii) The optical depth of the atmosphere due to aerosol scattering (say in a fairly polluted environment).

7. The aerosols within an aerosol haze all have the same size: 500 nm. The aerosols do not absorb solar radiation. $\lambda_{blue} = 470$ nm $\lambda_{red} = 640$ nm
(i) Will the haze look blue or red when illuminated by solar radiation? Explain as quantitatively as possible, with reference to the plot of scattering efficiency K_λ versus size parameter x shown with the formula sheet.
(ii) The haze is 500 m thick. The aerosol density is $N = 1000/cm^3$. What fraction of the incident blue light will pass directly through the haze without scattering?

red: 640 nm
blue: 470 nm
r = 500 nm
x(blue) = 2*pi*r/lambda = 3140/470 = 6.7
x(red) = 2*pi*r/lambda = 4.9
would look red
sigma = 4*pi*r2 = 3.14E-12
N = 1000/cm3 = 1E+09/m3
K 1.5
tau = N*K*sigma*ds = 2
ds = 2/(N*K*sigma) = 425 m

Chapter 4: Weighting Functions

1. We discussed weighting functions in class.
(i) Define in words what a weighting function is.
(ii) Weighting functions often have a peak at a particular altitude. Why? Make sure you explain why a weighting function would decrease above the peak, and decrease below the peak. A diagram may be helpful.
(iii) Weighting functions typically peak at a higher altitude when the mixing ratio of a gas is doubled. Explain. A diagram may be helpful.

2. Suppose that a CO_2 weighting function peaks in the mid-troposphere and goes to zero at the surface and in the upper atmosphere.
(i) Why would the weighting function go to zero near the surface?
(ii) Why would the weighting function go to zero in the upper atmosphere?

Chapter 4: Energy Balance Climate Models

1. Venus is an average distance of 1.08×10^{11} m from the sun. The energy output of the sun is roughly 3.86×10^{26} J/s (1 J/s = 1 W).
(i) What is the value of the solar flux S0 (in W/m2) at Venus?
(ii) The solar albedo A (also called solar reflectivity) of Venus is $A = 0.76$. What is the effective longwave emission temperature T_E of Venus? (Recall that you can also think of T_E as the average surface temperature of a planet if it had no atmosphere, and A was the solar reflectivity of the surface.)

2. A planet has the following properties: (i) there is no solar absorption in the atmosphere, (ii) the global 24-hour average, top of the atmosphere, incident solar flux is S0/4, (iii) the solar reflectivity of the planet is A, (iv) the atmosphere has a grey body emissivity ϵ, (v) the average surface temperature is T_s, (vi) the surface of the planet can be considered a black body, and (vi) the average atmospheric temperature is T_a.
(i) Write down an equation for the overall top of the atmosphere radiative energy balance (SW + LW) of the planet involving the variables S0, A, T_a, ϵ, and T_s (plus σ of course).
(ii) Write down an equation for the radiative energy balance (SW + LW) of the surface of the planet involving the variables S0, A, T_a, ϵ, and T_s.
(iii) Write down an equation for the radiative energy balance (SW + LW) of the atmosphere of the planet involving the variables T_a, ϵ, and T_s.
(iv) Derive an expression for the surface temperature T_s of the planet in terms of S0, A, and ϵ
(iv) The globally averaged surface temperature of Venus is 733 K. Assume the one layer model discussed in this question applies to Venus. Use the values of A and S0 for Venus from the previous question to estimate the longwave emissivity ϵ of the atmosphere on Venus. PROBLEM: get emissivity larger than 1, since Ts so big; model not really applicable.

3. The energy output of the sun is roughly 3.86×10^{26} J/s = W.
(i) Calculate the value of the solar constant at the earth on July 4, when the earth is 1.52×10^{11} m from the sun.
(ii) Assume that the total emission of thermal (long-wave) radiation from the earth can be written $A_e \sigma T_E^4$, where T_E is the effective emission temperature of the earth. Assume that the earth obeys an energy balance in which the only external source of energy is the absorption of solar radiation, and the only loss of energy is emission of thermal radiation. Assume that the solar albedo (reflectivity) of the earth is $A = 0.31$. Calculate the T_E of the earth on July 4.

(iii) On January 3, the earth is 1.47×10^{11} m from the sun. Calculate the difference between the T_E of the earth on July 4 and January 3.
Answer: (i) Solar flux at earth = (Power of Sun)/(area of sphere with radius Re). As the distance Re of the earth from the sun increases the output power of the sun gets distributed over a large area, and the solar flux goes down. Same type of question as how much warmth do you get r distance away from 100 W light bulb. (ii) Done in the notes. (iii) recalculate the solar flux as in (i) and find the new TE using same procedure as in (ii).

4. Suppose the value of the solar flux incident on a planet is $S_0 = 1370$ W/m^2 (i.e. the same as the earth). The atmosphere of the planet does not interact with SW or LW radiation. The surface of the planet absorbs all incident SW and LW radiation. The planet is completely covered by a cloud with LW emissivity $\epsilon = 0.6$, SW absorptivity $a = 0.1$, and SW reflectivity $R = 0.4$. Assume that the cloud, the planet as a whole, and the surface of the planet, are in radiative equilibrium. Let T_s and T_c refer to the temperatures of the surface and cloud respectively.
(i) Write down an energy balance equation for the cloud.
(ii) Write down an energy balance equation for the surface.
(iii) Write down an energy balance equation for the planet as a whole (i.e. top of the atmosphere energy balance).
(iv) Calculate T_s.
(v) Calculate T_c.

5. Make the following assumptions:
- The stratosphere is in global radiative equilibrium.
- The solar constant is $\mathrm{S}_0 = 1370$ W/m^2.
- The stratosphere absorbs 5 % of the incoming solar energy, ie. solar absorptivity $\mathrm{a}_s = 0.05$.
- The average temperature of the stratosphere is $T_s = 265$ K.
- The longwave radiation entering the stratosphere from the troposphere can be characterized by a single temperature $T_t = 250$ K.
- The stratosphere can be characterized by a single LW emissivity/absorptivity ϵ.
- There is no LW energy entering the stratosphere from above (space is too cold).
- There is no shortwave (solar) energy entering the stratosphere from below.
(i) What is the downward, global, 24-hour average solar flux at the top of the atmosphere?
(ii) Calculate the long wave emissivity ϵ of the stratosphere.

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SWin + LWin = SWout + LWout
(as/4)*S0 + eps*sigma*Tt**4 = 2*eps*sigma*Ts**4
(as/4)*S0 = 2*eps*sigma*Ts**4 - eps*sigma*Tt**4
(as/4)*S0 = eps*sigma*[2*Ts**4 - Tt**4]
eps = (as/4)*S0/sigma*[2*Ts**4 - Tt**4]
= 34.25/sigma*[4.93E09 - 2.80E09]
eps = 0.28
```

(iii) How are CO_2 increases expected to affect stratospheric temperatures? Explain with reference to your stratospheric energy budget.

6. It is expected that higher concentrations of greenhouse gases in the stratosphere will be associated with stratospheric cooling.
Suppose an air parcel in the stratosphere has a LW emissivity (or absorptivity) of ϵ, and a solar absorptivity of a. Assume also that the temperature of the ground is T_g, that the upwelling LW radiation from the ground is not absorbed by the atmosphere (not such a good assumption in the stratosphere), and that the local temperature is T_{strat}.

The local energy balance can be written:

$$\text{SW absorbed } + \text{ LW absorbed } = \text{ LW emitted}$$

$$a(S_0/4) + \epsilon\sigma T_g^4 = 2\epsilon\sigma T_{strat}^4$$

The factor of two in the emission is because emission occurs up and down, whereas absorption is only from below (in reality there would be a slight amount of absorption from above since there would be some greenhouse gases above the air parcel). The incident solar radiation will vary over a day. S_0 is divided by 4 to give a 24 hour average.
Solving for T_{strat}:

$$T_{strat}^4 = T_g^4/2 + (aS_0)/(8\epsilon\sigma)$$

An increase in the concentration of greenhouse gases would give rise to an increase in ϵ. This would decrease the second term on the right, giving rise to a decrease in T_{strat}. Note that if the solar absorptivity $a = 0$, then T_{strat} is independent of ϵ.
The energy balance given above for the stratosphere would also apply to the troposphere. However, the solar absorptivity of the troposphere is quite weak, so that one would replace the solar heating by condensational heating from rainfall. However, the troposphere would not be expected to cool in response to an increase in ϵ because increased LW emission will be balanced by increased condensational heating (i.e. increased rainfall).

7. (i) The sun is at an angle of $\theta = 60^o$ from the vertical. The solar constant is 1370 W/m^2. Assume there is no absorption or scattering of solar radiation in the atmosphere. What is the downward solar flux at the surface? (W = J/s)
(ii) Assume that the surface temperature is 290 K. What is the upward LW flux at the top of the atmosphere? Assume there is no absorption or emission of LW radiation in the atmosphere.
(iii) Assume that a cloud is inserted into the atmosphere at 10 km, that its temperature is 240 K, and that its emissivity $\epsilon = 0.3$. Calculate the upward flux of LW radiation at the top of the atmosphere. Assume that there is no absorption or emission of LW radiation in the atmosphere other than by the cloud.
(iv) Assume that the cloud has a solar (SW) reflectivity A = 0.3. It does not absorb solar radiation. Calculate the change in net change in downward radiation (SW + LW) at the surface due to the introduction of the cloud. (You can assume that the surface absorbs all incident solar radiation. This avoids multiple reflections of solar radiation between the ground and the cloud).

8. Make the following assumptions:
- The stratosphere is in global radiative equilibrium.
- The solar constant is S_0 = 1370 W/m^2.
- The stratosphere absorbs 10 % of the incoming solar energy, ie. solar absorptivity $a_s = 0.1$.
- The average temperature of the stratosphere is $T_s = 265$ K.
- The longwave radiation entering the stratosphere from the troposphere can be characterized by a single temperature $T_t = 230$ K.
- There is no shortwave (solar) energy entering the stratosphere from below.
- The stratosphere can be characterized by a single LW emissivity/absorptivity ϵ.
- There is no LW energy entering the stratosphere from above (space is too cold).

(i) Estimate the long wave emissivity ϵ of the atmosphere.

(ii) Assume that the only LW absorber in the stratosphere is CO_2, and that a doubling of the concentration of CO_2 in the stratosphere increases the emissivity by 50 %. That is, the new emissivity is 1.5 times the previous emissivity. Estimate the new temperature of the stratosphere.

Chapter 10: Climate Dynamics (T10)

1. Specify and briefly discuss two climate feedbacks in the earth's climate. For each feedback, specify whether the feedback is positive or negative, and explain why it is positive or negative.

2. When is net ice sheet growth favored? Under conditions of cooler summers + warmer winters, or under conditions of warmer summers + cooler winters? Explain. A diagram may be helpful in your explanation.

3. (i) Show a rough plot of the seasonal variation of the area of the Arctic Ocean covered by sea ice (from January to December). Label the months of maximum and minimum sea ice cover.
(ii) During what month have the negative trends in Arctic sea ice area over the past several decades been largest?

4. (i) Which climate feedback gives rise to the largest uncertainty in future climate projections?
(ii) Why is this climate feedback so uncertain?

5. (i) Define the term "radiative forcing".
(ii) Give an example of a radiative forcing. How does it affect the radiative budget of the earth?

6. Currently, ice ages are driven by orbital forcings. These forcings also existed 50 million years ago. However, ice ages did not occur. Why not?

7. Suppose that the NAO is in the high index phase (deeper than normal sea level pressure over Iceland). It is winter.
(i) How would you expect this phase of the NAO to affect surface temperatures over Europe? Explain the reason for this change.
Answer: Europe warms mainly because of increased surface flow over North Atlantic, and increased heat and moisture fluxes. This warmer air then advected over Europe. Could also mention northward movement of jetstream.
(ii) How would you expect this phase of the NAO to affect rainfall patterns over Europe? Explain the reason for this change.

8. A location in the northern Hemisphere has a "Mediterranean" climate. Plot how you would expect temperature and rainfall to vary over the course of a year (January to December) at this location.

9. The imbalance in the total radiative flux at the top of the atmosphere is estimated to be 0.9 W/m^2. (i.e. the earth absorbs 0.9 W/m^2 more incoming radiative energy than leaves the earth). Estimate the sea level trend that would occur in mm/year if all of this surplus radiative energy were used to melt ice on land, which then all drained into the ocean. Assume that the oceans cover 80 % of the earth's surface. The radius of the earth is $R_e = 6.37 \times 10^6$ m (can do this question without using R_e however).

10. At present, the imbalance in the total radiative flux density at the top of the atmosphere due to an increasing storage of energy in the Earth system in response to the buildup of greenhouse gases is estimated to be 0.8 W/m^2. (i.e. the earth absorbs 0.8 W/m^2 more SW and LW energy than it reflects or emits to space.) This energy is going into heating the atmosphere, oceans, soil, and

toward melting the ice sheets. Assume that 10 % of this radiative imbalance is going into melting the Antarctic and Greenland ice sheets. The total volume of the two ice sheets is 42×10^{15} m^3. The radius of the earth is $R_e = 6.37 \times 10^6$ m.
(i) How long would it take to entirely melt the two ice sheets under these assumptions? Give your answer in years.
(ii) Estimate the amount of sea level rise that would occur in 100 years due to the melting of the two ice sheets under these assumptions. Give your answer in m.
Answer: The Radius of the earth Re = 6.37 106 m. Get J/s energy = 4E13 J/s from 10 % of 0.8 W/m2. Divide by heat of melting to get kg ice melted/s = 1.2E8 kg/s. Mass of two ice sheets is 4.2E19 kg. They would melt in 3.5E11 s This gives 11,098 years to entirely melt. Get 0.74 m for sea level rise in 100 years.

11. In regions with permafrost, the soil temperature varies with depth in a way which is different from other regions. Plot a typical variation of soil temperature with depth at a location with permafrost. Show a profile at the end of winter (e.g. April) and a profile at the end of summer (e.g. September). The vertical axis should be depth below the surface while the horizontal axis should refer to temperature.

12. (i) Show a rough plot of the seasonal variation of the area of the Arctic Ocean covered by sea ice (from January to December).
(ii) During what month are the negative trends in Arctic sea ice area over the past several decades largest?

13. (i) Give an example of climate variability forced by a boundary condition.
(ii) Give an example of climate variability that is internally generated within the atmosphere.

14. The El Nino Southern Oscillation (ENSO) is sometimes described as an example of “air-sea interaction”. Explain.