ATCAT SEMINAR – 18 janvier 2022

Monoidal Closed but not Compact Closed

Summary:

- Compact Closed categories are monoidal closed
- self-dual compact closed categories,
  $$A \multimap B = A \otimes B$$
- What about the converse?

  If I have a monoidal closed category where $A \multimap B = A \otimes B$, is it a self-dual compact closed category?

The answer seems to be NO ....
But what's the counter example???
WANT: COUNTER-EXAMPLE!

Where this problem came from:

Cartesian Reverse Differential Category

$\cup|$

Linear Maps

Linear Logic version is compact closed

only been able to show monoidal closed

$A \multimap B = A \otimes B$

Monoidal Closed:

$\mathbb{X}$ symmetric monoidal category

$\otimes$ - tensor $\qquad I$ - unit

strict

Closed: $A \otimes - \dashv A \multimap -$

$$A \otimes (A \multimap X) \xrightarrow{\epsilon} X$$

evaluation map

$$\frac{A \otimes X \xrightarrow{F} Y}{X \xrightarrow{\lambda F} A \multimap Y}$$

$$X \xrightarrow{\eta} A \multimap (A \otimes X)$$

$$\begin{tikzcd}
A \otimes X \arrow[r, "F"] \arrow[d, "1 \otimes \lambda F"'] & Y \\
A \otimes (A \multimap Y) \arrow[ur, "\epsilon"'] &
\end{tikzcd}$$

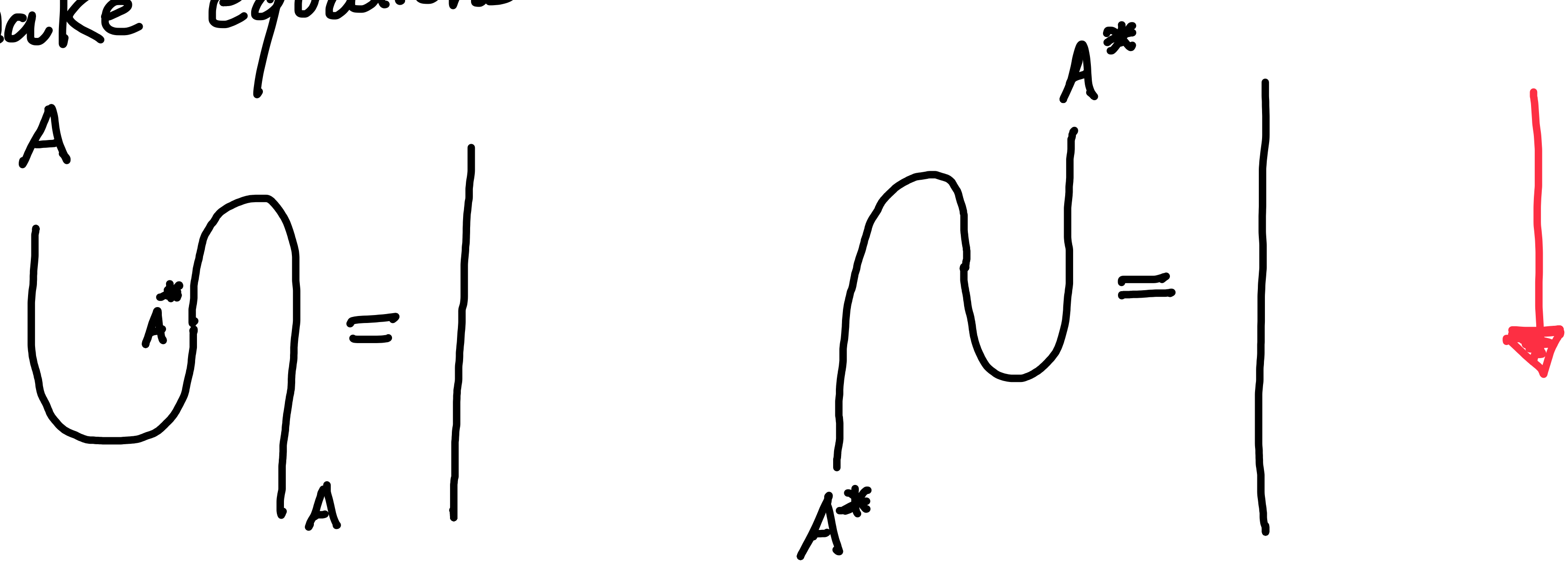
$$\begin{tikzcd}
X \arrow[r, "\lambda F"] \arrow[d, "\eta"'] & A \multimap Y \\
A \multimap (A \otimes X) \arrow[ur, "A \multimap F"'] &
\end{tikzcd}$$

Compact Closed:

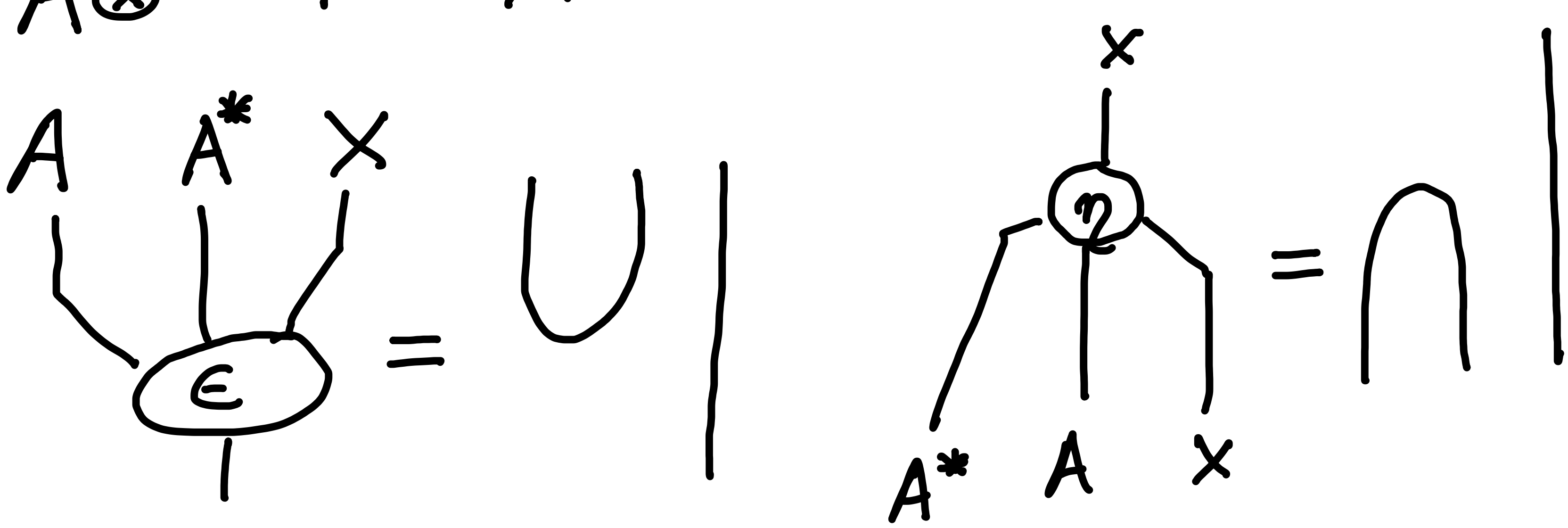
symmetric monoidal category where every object A has a dual A^*

$\cap : I \longrightarrow A^* \otimes A \qquad \cup : A \otimes A^* \longrightarrow I$

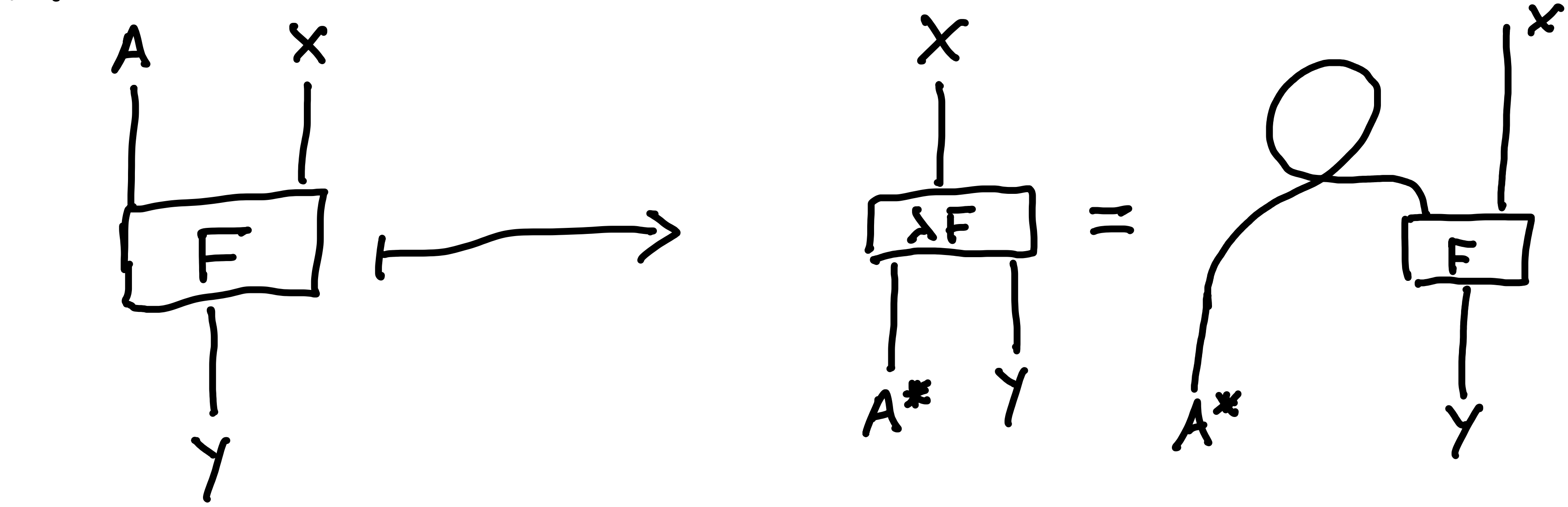
Snake equations:



$A \otimes - \vdash A^* \otimes -$



Monoidal Closed: $A \multimap B = A^* \otimes B$

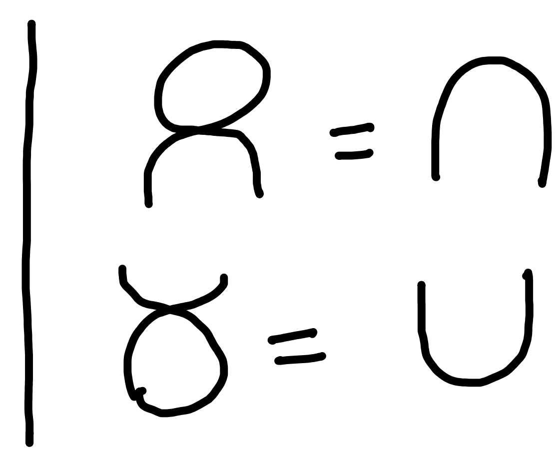


Self-dual Compact Closed Categories

where A is it's own dual:

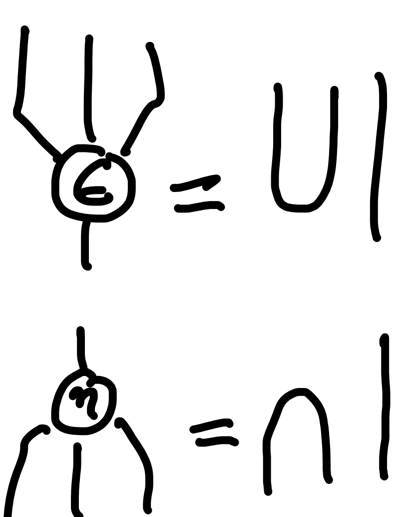
$A^* = A$

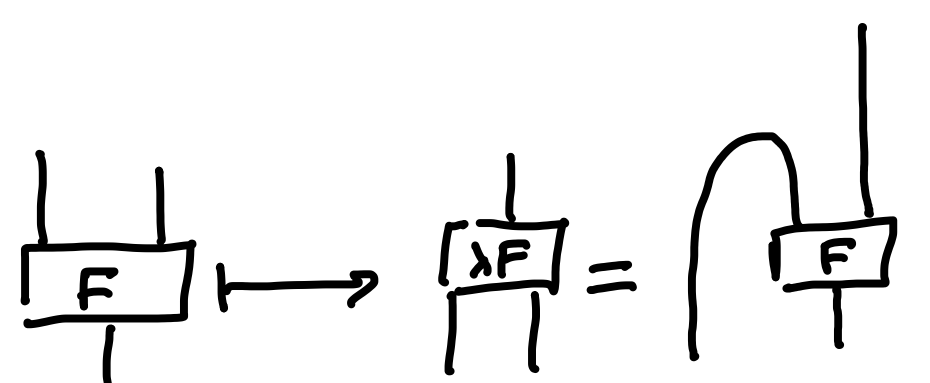
and also that:



$\cap : I \longrightarrow A \otimes A$

$\cup : A \otimes A \longrightarrow I$





$A \otimes - \vdash A \otimes -$

What about the converse?

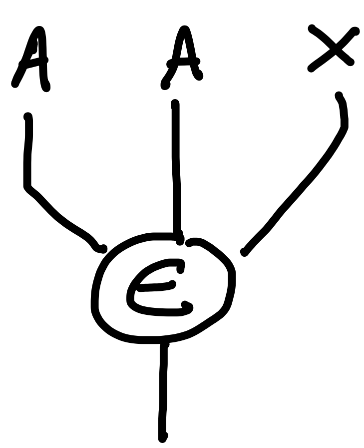
Monoidal closed category:

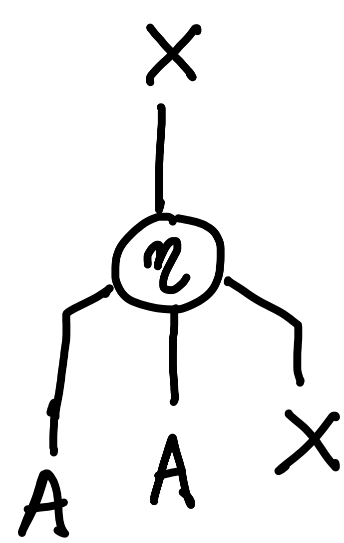
$$A \multimap - \;=\; A \otimes -$$

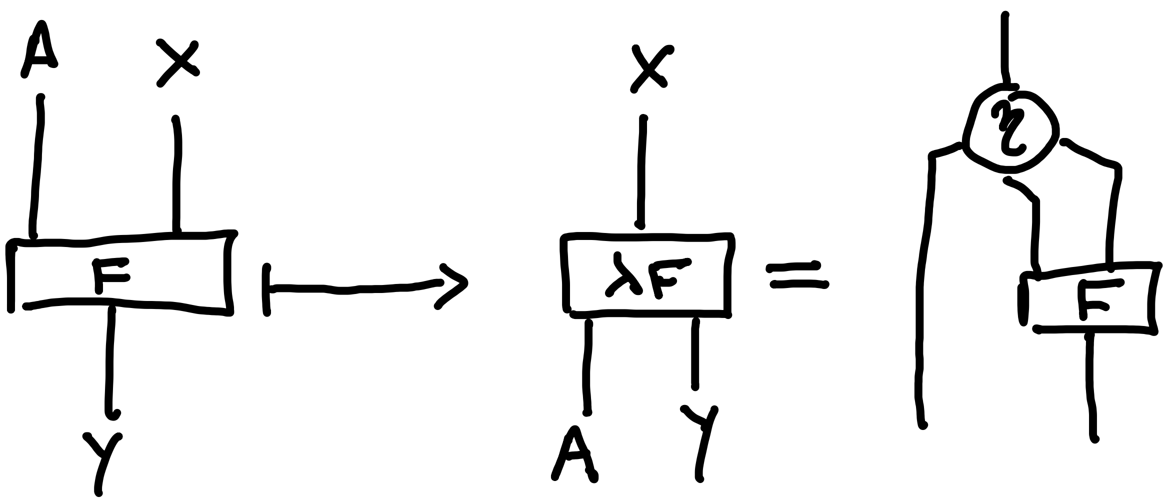
A, B are dual $\Longrightarrow$ $A \otimes - \vdash B \otimes -$

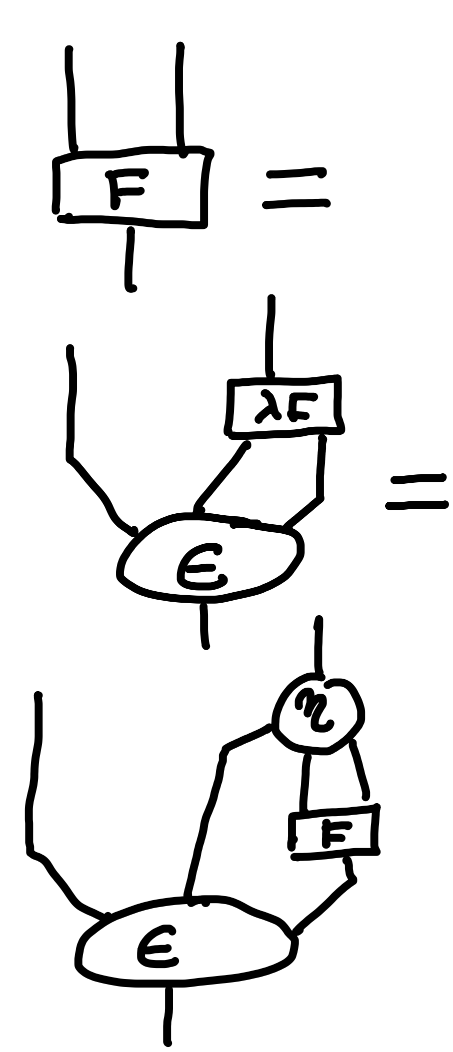
$A \otimes - \vdash B \otimes - \Longrightarrow$ dual? NO

$$A \otimes - \vdash A \otimes -$$

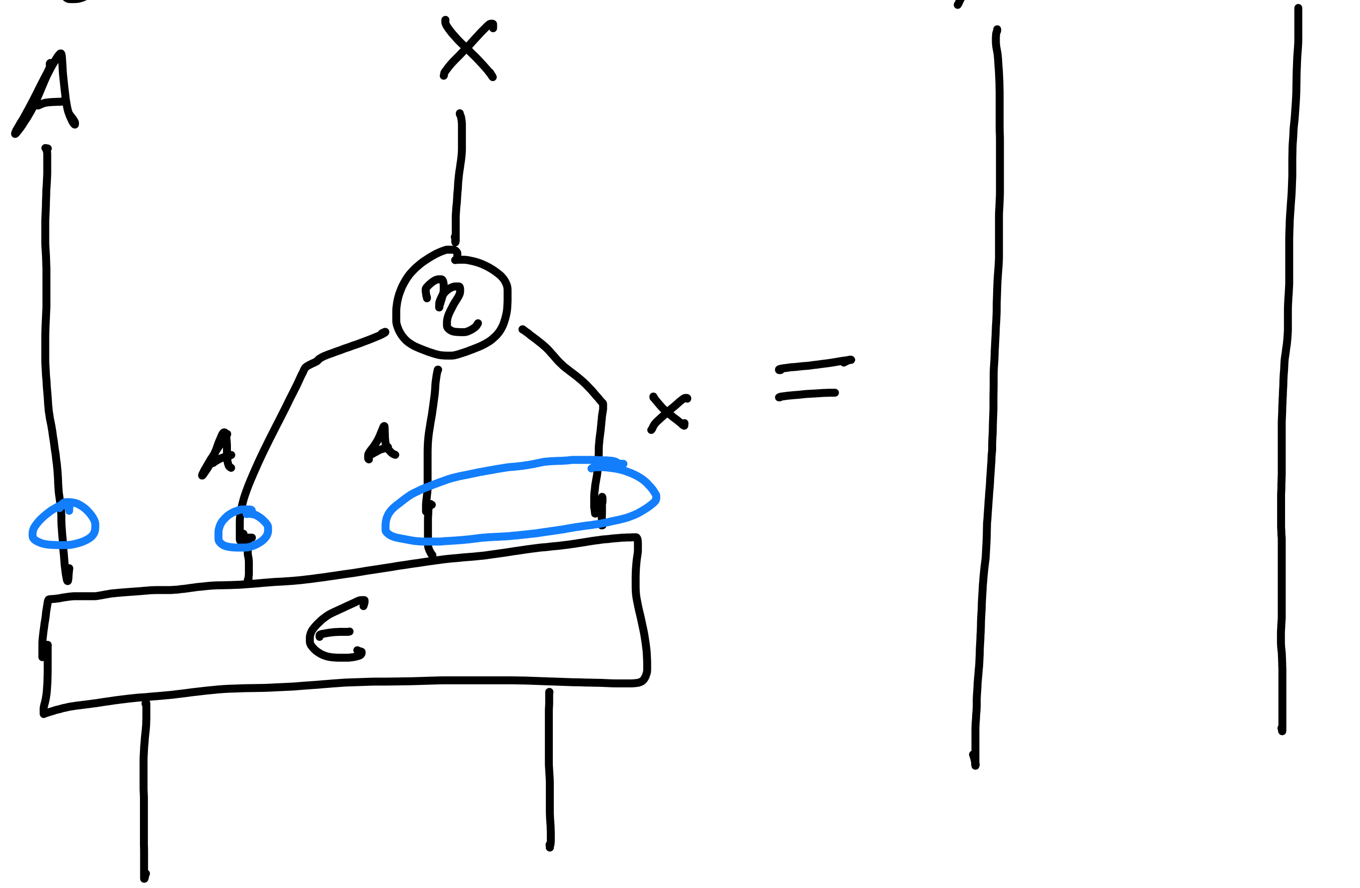


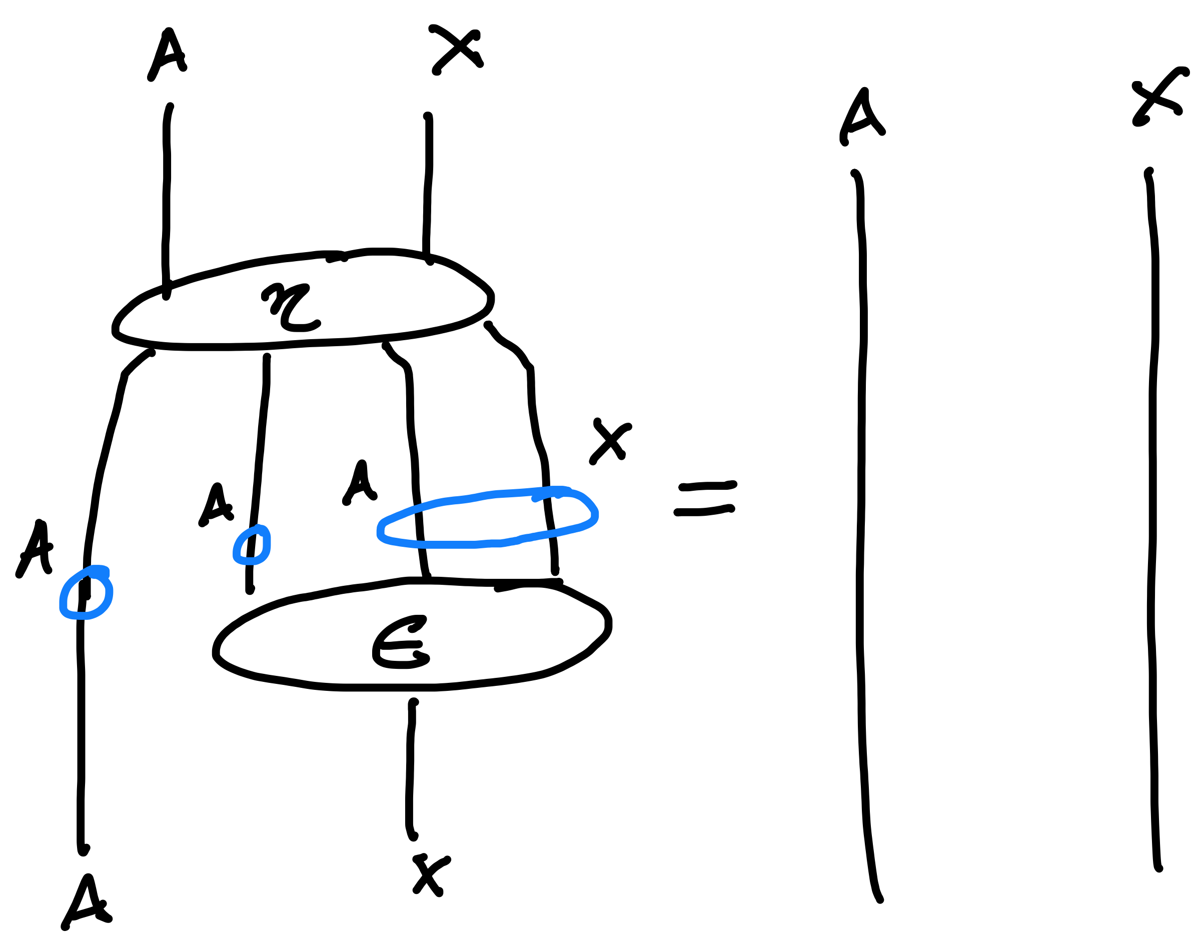




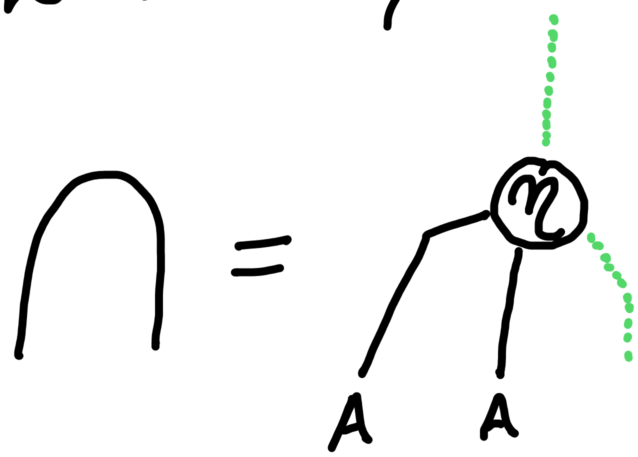


Adjoint:

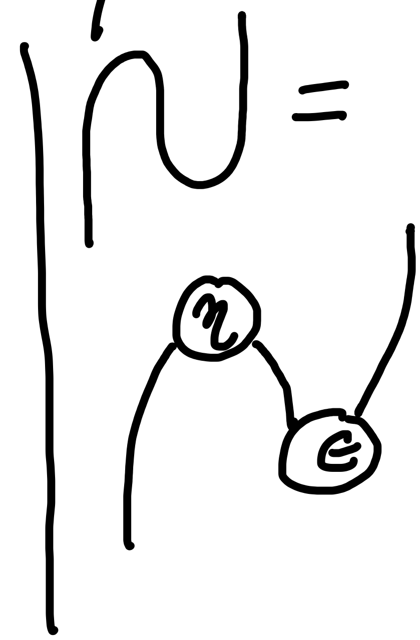




Let's try and build cups/caps:

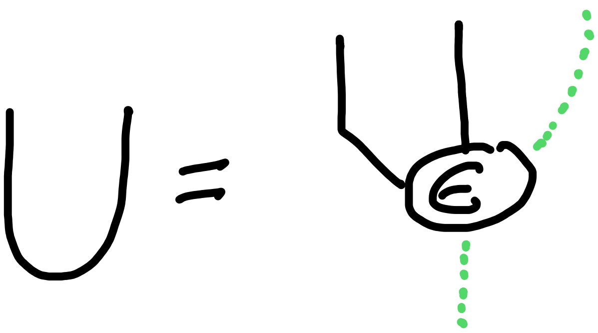


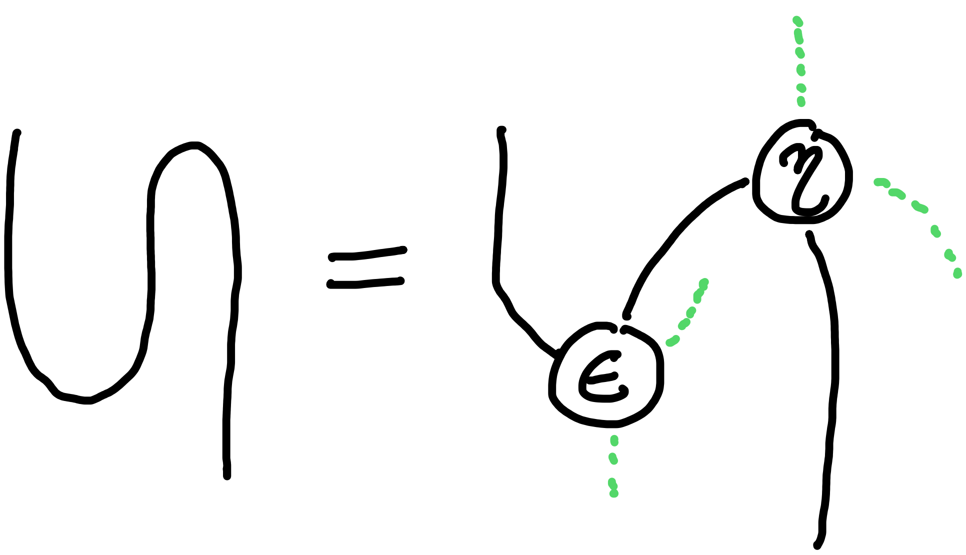
η_I $x = I$



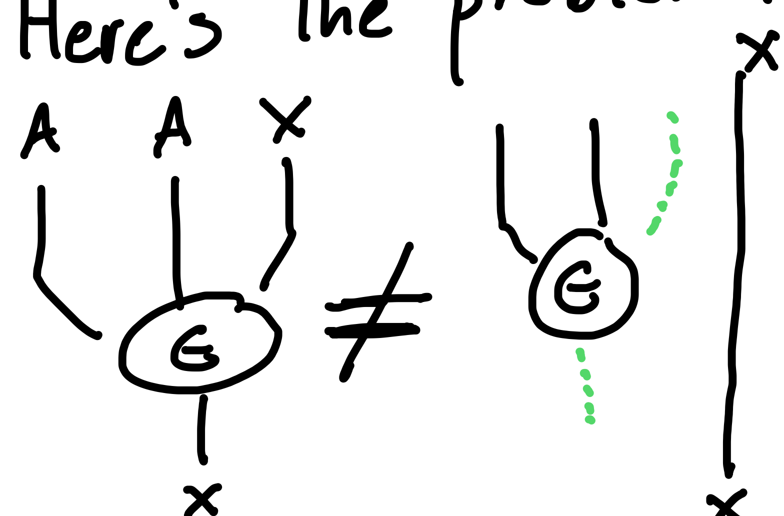
$x = I$

ϵ_I





Here's the problem:



compact closed case: