52.

Let A_1 be the event that #1 fails and A_2 be the event that #2 fails. We assume that $P(A_1) = P(A_2) = q$ and that $P(A_1 \mid A_2) = P(A_2 \mid A_1) = r$. Then one approach is as follows:

$P(A_1 \cap A_2) = P(A_2 \mid A_1) \bullet P(A_1) = rq = .01$

$P(A_1 \cup A_2) = P(A_1 \cap A_2) + P(A_1' \cap A_2) + P(A_1 \cap A_2') = rq + 2(1-r)q = .07$

These two equations give 2q - .01 = .07, from which q = .04 and r = .25. Alternatively, with $t = P(A_1' \cap A_2) = P(A_1 \cap A_2')$, t + .01 + t = .07, implying t = .03 and thus q = .04 without reference to conditional probability

54.

$P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$

a. $P(A_2 \mid A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{.11}{.22} = .50$

b. $P(A_2 \cap A_3 \mid A_1) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.01}{.22} = .0455$

c.

$$P(A_2 \cup A_3 \mid A_1) = \frac{P[A_1 \cap (A_2 \cup A_3)]}{P(A_1)} = \frac{P[(A_1 \cap A_2) \cup (A_1 \cap A_3)]}{P(A_1)}$$

$$= \frac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.15}{.22} = .682$$

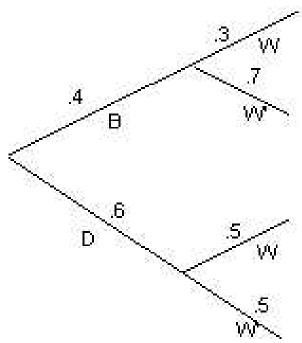
d.

$$P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.01}{.53} = .0189$$

This is the probability of being awarded all three projects given that at least one project was awarded.

62. Using a tree diagram, B = basic, D = deluxe, W = warranty purchase, W' = no warranty

We want P(B|W) = $\dfrac{P(B \cap W)}{P(W)} = \dfrac{.12}{.30 + .12} = \dfrac{.12}{.42} = .2857$



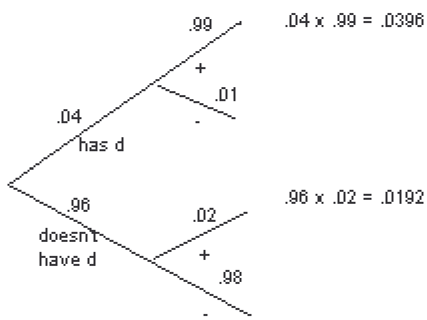
$.4 \times .3 = .12 = P(B \cap W)$

$.4 \times .7 = .28 = P(B \cap W')$

$.6 \times .5 = .30 = P(D \cap W)$

$.6 \times .5 = .30 = P(D \cap W')$

64.



e. P(+) = .0588

f. P(has d | +) = $\dfrac{.0396}{.0588} = .6735$

g. P(doesn't have d | -) = $\dfrac{.9408}{.9412} = .9996$

72.

Using subscripts to differentiate between the selected individuals,

$P(O_1 \cap O_2) = P(O_1) \bullet P(O_2) = (.44)(.44) = .1936$

P(two individuals match) = $P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2)$

$= .42^2 + .10^2 + .04^2 + .44^2 = .3816$

78.

P(system works) = P(1 – 2 works $\cup$ 3 – 4 works)

= P(1 – 2 works) + P(3 – 4 works) - P(1 – 2 works $\cap$ 3 – 4 works)

= P(1 works $\cup$ 2 works) + P(3 works $\cap$ 4 works) – P(1 – 2) • P(3 – 4)

= (.9+.9-.81) + (.9)(.9) – (.9+.9-.81)(.9)(.9)

= .99 + .81 - .8019 = .9981

102.

Let B denote the event that a component needs rework. Then

$$P(B) = \sum_{i=1}^{3} P(B| A_i) \cdot P(A_i) = (.05)(.50) + (.08)(.30) + (.10)(.20) = .069$$

Thus $P(A_1 | B) = \dfrac{(.05)(.50)}{.069} = .362$

$$P(A_2 | B) = \frac{(.08)(.30)}{.069} = .348$$

$$P(A_3 | B) = \frac{(.10)(.20)}{.069} = .290$$