Assignment 6 :

Chapter 3: Questions 64, 68, 70, 74, 78, 80, 84

64.

a. X ~ Hypergeometric N=15, n=5, M=6

b.

P(X=2) = $\frac{\binom{6}{2}\binom{9}{3}}{\binom{15}{5}} = \frac{840}{3003} = .42$

P(X≤2) = P(X=0) + P(X=1) + P(X=2) =0.713

P(X≥2) = 1 – P(X≤1) = 1 – [P(X=0) + P(X=1)] = $1 - \frac{126+756}{3003} = .706$

c. E(X) = $5\left(\frac{6}{15}\right) = 2$; V(X) = $\left(\frac{15-5}{14}\right)\cdot 5\cdot\left(\frac{6}{15}\right)\cdot\left(1-\frac{6}{15}\right) = .857$;

$\sigma = \sqrt{V(X)} = .926$

68.

a. h(x; 6,4,11)

b. $6\cdot\left(\frac{4}{11}\right) = 2.18$

70.

a. h(x;10,15,50)

b. When N is large relative to n, h(x;n,M,N) $\doteq b\left(x;n,\frac{M}{N}\right)$,

so h(x;10,150,500) $\doteq b(x;10,.3)$

c. Using the hypergeometric model, E(X) = $10\cdot\left(\frac{150}{500}\right) = 3$ and

V(X) = $\frac{490}{499}(10)(.3)(.7) = .982(2.1) = 2.06$

Using the binomial model, E(X) = (10)(.3) = 3, and

V(X) = 10(.3)(.7) = 2.1

74. **If t**he interpretation of “roll” here is a pair of tosses of a single player’s die(two tosses by A or two by B). With S = doubles on a particular roll, p = $\frac{1}{6}$. Furthermore, A and B are really identical (each die is fair), so we can equivalently imagine A rolling until 10 doubles appear. The P(x rolls) = P(9 doubles among the first x – 1 rolls and a double on the x^{th} roll =

$$\binom{x-1}{9}\left(\frac{5}{6}\right)^{x-10}\left(\frac{1}{6}\right)^{9}\cdot\left(\frac{1}{6}\right)=\binom{x-1}{9}\left(\frac{5}{6}\right)^{x-10}\left(\frac{1}{6}\right)^{10}$$

E(X-10) = $\frac{r(1-p)}{p}=\frac{10(\frac{5}{6})}{\frac{1}{6}}=10(5)=50$, E(X)=60

V(X) = $\frac{r(1-p)}{p^2}=\frac{10(\frac{5}{6})}{\left(\frac{1}{6}\right)^2}=10(5)(6)=300$

If the interpretation of "roll" here is one toss of a single player's die(two tosses together by A and B). With S = doubles on a particular roll.

The chance that A & B roll a "double" is p = 6/36 = 1/6 and successive pairs of rolls are independent.

So, we can write X = 5 + Y, where Y ~ nb(r=5,p=1/6). (Y is the number of non-doubles rolled prior to the 5^{th} double.) Substitute y = x-5 into the negative binomial pmf to find p(x) =

$$\binom{x-5+5-1}{5-1}\left(\frac{1}{6}\right)^{5}\left(\frac{5}{6}\right)^{x-5}=\binom{x-1}{4}\left(\frac{1}{6}\right)^{5}\left(\frac{5}{6}\right)^{x-5}\quad \text{for } x = 5, 6, \ldots$$

E(X) = 5 + E(Y) = 5 + $\frac{5(\frac{5}{6})}{\frac{1}{6}}$ = 5 + 25 = 30, V(X) = V(Y) = $\frac{5(\frac{5}{6})}{\left(\frac{1}{6}\right)^2}$ = 150.

78.

a. $P(X = 1) = F(1;2) - F(0;2) = .982 - .819 = .163$

b. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - F(1;2) = 1 - .982 = .018$

c. $P(1^{st} \text{ doesn't} \cap 2^{nd} \text{ doesn't}) = P(1^{st} \text{ doesn't}) \cdot P(2^{nd} \text{ doesn't})$
$= (.819)(.819) = .671$

80.

a. The experiment is binomial with n = 10,000 and p = .001,
so $\mu = np = 10$ and $\sigma = \sqrt{npq} = 3.161$.

b. X has approximately a Poisson distribution with $\lambda = 10$,
so $P(X > 10) \approx 1 - F(10;10) = 1 - .583 = .417$

c. $P(X = 0) \approx 0$

84. Let X = the number of diodes on a board that fail.

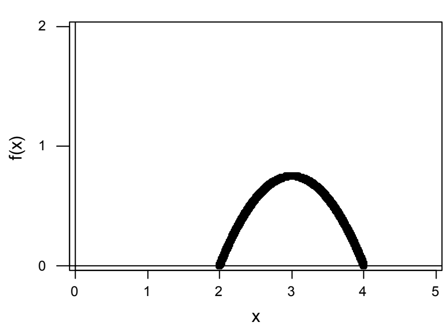
a. $E(X) = np = (200)(.01) = 2$, $V(X) = npq = (200)(.01)(.99) = 1.98$, $\sigma_X = 1.407$

b. X has approximately a Poisson distribution with $\lambda = np = 2$,
so $P(X \geq 4) = 1 - P(X \leq 3) = 1 - F(3;2) = 1 - .857 = .143$

c. P(board works properly) = P(all diodes work) = $P(X = 0) = F(0;2) = .135$
Let Y = the number among the five boards that work, a binomial r.v. with n = 5 and p = .135.
Then $P(Y \geq 4) = P(Y = 4) + P(Y = 5) = \binom{5}{4}(.135)^4(.865) + \binom{5}{5}(.135)^5(.865)^0 = .00144 + .00004 = .00148$

Chapter 4: Questions 6, 8, 18, 22

6.



a.

b. 1 = $\int_2^4 k[1-(x-3)^2]dx = \int_{-1}^1 k[1-u^2]du = \frac{4}{3} \Rightarrow k = \frac{3}{4}$

c. P(X > 3) = $\int_3^4 \frac{3}{4}[1-(x-3)^2]dx = .5$ by symmetry of the p.d.f

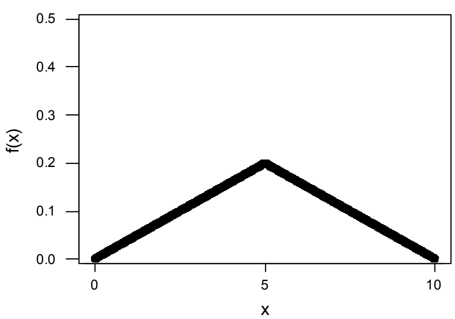
d. $P\left(\frac{11}{4} \le X \le \frac{13}{4}\right) = \int_{11/4}^{13/4} \frac{3}{4}[1-(x-3)^2]dx = \frac{3}{4}\int_{-1/4}^{1/4}[1-(u)^2]du = \frac{47}{128} \approx .367$

e. P(|X-3| > .5) = 1 – P(|X-3| ≤ .5) = 1 – P(2.5 ≤ X ≤ 3.5)

= 1 - $\int_{-.5}^{.5} \frac{3}{4}[1-(u)^2]du = \frac{5}{16} \approx .313$

8.

a.



b. $\int_{-\infty}^{\infty} f(y)dy = \int_0^5 \frac{1}{25}ydy + \int_5^{10}(\frac{2}{5} - \frac{1}{25}y)dy$ = $\left.\frac{y^2}{50}\right]_0^5 + \left.\left(\frac{2}{5}y - \frac{1}{50}y^2\right)\right]_5^{10}$

= $\frac{1}{2} + \left[(4-2)-(2-\frac{1}{2})\right] = \frac{1}{2} + \frac{1}{2} = 1$

c. P(Y ≤ 3) = $\int_0^3 \frac{1}{25}ydy = \left.\frac{y^2}{50}\right]_0^5 = \frac{9}{50} \approx .18$

d. P(Y ≤ 8) = $= \int_0^5 \frac{1}{25}ydy + \int_5^8(\frac{2}{5} - \frac{1}{25}y)dy = \frac{23}{25} \approx .92$

e. P(3 ≤ Y ≤ 8) = P(Y ≤ 8) - P(Y < 3) = $\frac{46}{50} - \frac{9}{50} = \frac{37}{50} = .74$

f. P(Y < 2 or Y > 6) = $= \int_0^3 \frac{1}{25} y\,dy + \int_6^{10} (\frac{2}{5} - \frac{1}{25} y)\,dy = \frac{2}{5} = .4$

18.

a. F(X) = $\frac{x-A}{B-A}$ = p ⇒ x = (100p)th percentile = A + (B - A)p

b.

$$E(X) = \int_A^B x \cdot \frac{1}{B-A}\,dx = \frac{1}{B-A} \cdot \frac{x^2}{2}\Big]_A^B = \frac{1}{2} \cdot \frac{1}{B-A} \cdot \left(B^2 - A^2\right) = \frac{A+B}{2}$$

$$E(X^2) = \frac{1}{3} \cdot \frac{1}{B-A} \cdot \left(B^3 - A^3\right) = \frac{A^2 + AB + B^2}{3}$$

$$V(X) = \left(\frac{A^2 + AB + B^2}{3}\right) - \left(\frac{(A+B)}{2}\right)^2 = \frac{(B-A)^2}{12}, \quad \sigma_x = \frac{(B-A)}{\sqrt{12}}$$

c.

$$E(X^n) = \int_A^B x^n \cdot \frac{1}{B-A}\,dx = \frac{B^{n+1} - A^{n+1}}{(n+1)(B-A)}$$

22.

a. For $1 \le x \le 2$, F(x) = $\int_1^x 2\left(1 - \frac{1}{y^2}\right)dy = 2\left(y + \frac{1}{y}\right)\Big]_1^x = 2\left(x + \frac{1}{x}\right) - 4,$ so

$$F(x) = \begin{cases} 0 & x < 1 \\ 2\left(x + \frac{1}{x}\right) - 4 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

b. $2\left(x_p + \frac{1}{x_p}\right) - 4 = p$ ⇒ $2x_p^2 - (4 - p)x_p + 2 = 0$ ⇒ $x_p = \frac{1}{4}[4 + p + \sqrt{p^2 + 8p}]$ To find $\tilde{\mu}$, set p = .5 ⇒ $\tilde{\mu}$ = 1.64

c. E(X) = $\int_1^2 x \cdot 2\left(1 - \frac{1}{x^2}\right)dx = 2\int_1^2 \left(x - \frac{1}{x}\right)dx = 2\left(\frac{x^2}{2} - \ln(x)\right)\Big]_1^2 = 1.614$

E(X^2) = $2\int_1^2 \left(x^2 - 1\right)dx = 2\left(\frac{x^3}{3} - x\right)\Big]_1^2 = \frac{8}{3}$ ⇒ Var(X) = .0626

d. Amount left = max(1.5 – X, 0), so

E(amount left) = $$\int_1^2 \max(1.5 - x, 0) f(x)dx = 2\int_1^{1.5} (1.5 - x)\left(1 - \frac{1}{x^2}\right)dx = .061$$