Assignment 7:

Chapter 4: Questions 28, 30, 36, 38, 48, 50

28.

a.
$$\Phi(c) = .9100 \implies c \approx 1.34$$
 (.9099 is the entry in the 1.3 row, .04 column)

b.
$$9^{th}$$
 percentile = -91^{st} percentile = -1.34

c. $\Phi(c) = .7500 \implies c \approx .675$ since .7486 and .7517 are in the .67 and .68 entries, respectively.

d.
$$25^{\text{th}} = -75^{\text{th}} = -.675$$

e. $\Phi(c) = .06 \implies c \approx .-1.555$ (both .0594 and .0606 appear as the -1.56 and -1.55 entries, respectively).

30.

a.
$$P(X \le 100) = P\left(z \le \frac{100 - 80}{10}\right) = P(Z \le 2) = \Phi(2.00) = .9772$$

$$P\left(z \le \frac{80 - 80}{10}\right) = P(Z \le 0) = \Phi(0.00) = .5$$

$$P\left(\frac{65-80}{10} \le z \le \frac{100-80}{10}\right) = P(-1.50 \le Z \le 2)$$

$$= \Phi(2.00) - \Phi(-1.50) = .9772 - .0668 = .9104$$

d.
$$P(70 \le X) = P(-1.00 \le Z) = 1 - \Phi(-1.00) = .8413$$

e.
$$P(85 \le X \le 95) = P(.50 \le Z \le 1.50) = \Phi(1.50) - \Phi(.50) = .2417$$

f.
$$P(|X - 80| \le 10) = P(-10 \le X - 80 \le 10) = P(70 \le X \le 90)$$

 $P(-1.00 \le Z \le 1.00) = .6826$

36.
$$\mu = 43$$
; $\sigma = 4.5$

a.
$$P(X < 40) = P\left(z \le \frac{40 - 43}{4.5}\right) = P(Z < -0.667) = .2514$$
$$P(X > 60) = P\left(z > \frac{60 - 43}{4.5}\right) = P(Z > 3.778) \approx 0$$

b.
$$43 + (-0.67)(4.5) = 39.985$$

38. From Table A.3, P(-1.96 \le Z \le 1.96) = .95. Then P(\mu - .1 \le X \le \mu + .1) =
$$P\left(\frac{-.1}{\sigma} < z < \frac{.1}{\sigma}\right)$$
 implies that $\sigma = 1.96$, and thus that $\sigma = \frac{.1}{1.96} = .0510$

a.
$$P(20 - .5 \le X \le 30 + .5) = P(19.5 \le X \le 30.5) = P(-1.1 \le Z \le 1.1) = .7286$$

b.
$$P(\text{at most } 30) = P(X \le 30 + .5) = P(Z \le 1.1) = .8643.$$
 $P(\text{less than } 30) = P(X < 30 - .5) = P(Z < .9) = .8159$

50.
$$P = .10$$
; $n = 200$; $np = 20$, $npq = 18$

$$\mathbf{a.} \quad P(X \le 30) = \Phi\left(\frac{30 + .5 - 20}{\sqrt{18}}\right) = \Phi(2.47) = .9932$$

$$\mathbf{b.} \quad P(X < 30) = P(X \le 29) = \Phi\left(\frac{29 + .5 - 20}{\sqrt{18}}\right) = \Phi(2.24) = .9875$$

c.
$$P(15 \le X \le 25) = P(X \le 25) - P(X \le 14) = \Phi\left(\frac{25 + .5 - 20}{\sqrt{18}}\right) - \Phi\left(\frac{14 + .5 - 20}{\sqrt{18}}\right)$$

 $\Phi(1.30) - \Phi(-1.30) = .9032 - .0968 = .8064$

Chapter 1: 10, 18

a. Minitab generates the following stem-and-leaf display of this data:

```
5 | 9
6 | 33588
7 | 00234677889
8 | 127
9 | 077 | stem: ones
10 | 7 | leaf: tenths
11 | 368
```

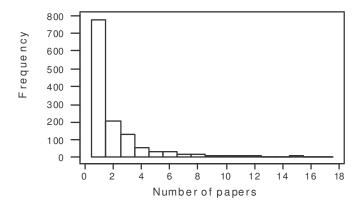
What constitutes large or small variation usually depends on the application at hand, but an often-used rule of thumb is: the variation tends to be large whenever the spread of the data (the difference between the largest and smallest observations) is large compared to a representative value. Here, 'large' means that the percentage is closer to 100% than it is to 0%. For this data, the spread is 11 - 5 = 6, which constitutes 6/8 = .75, or, 75%, of the typical data value of 8. Most researchers would call this a large amount of variation.

- **b.** The data display is not perfectly symmetric around some middle/representative value. There tends to be some positive skewness in this data.
- **c.** In Chapter 1, outliers are data points that appear to be *very* different from the pack. Looking at the stem-and-leaf display in part (a), there appear to be no outliers in this data. (Chapter 2 gives a more precise definition of what constitutes an outlier).
- **d.** From the stem-and-leaf display in part (a), there are 4 values greater than 10. Therefore, the proportion of data values that exceed 10 is 4/27 = .148, or, about 15%.

18.

a.

The following histogram was constructed using Minitab:



The most interesting feature of the histogram is the heavy positive skewness of the data.

Note: One way to have Minitab automatically construct a histogram from grouped data such as this is to use Minitab's ability to enter multiple copies of the same number by typing, for example, 784(1) to enter 784 copies of the number 1. The frequency data in this exercise was entered using the following Minitab commands:

```
MTB > set c1

DATA> 784(1) 204(2) 127(3) 50(4) 33(5) 28(6) 19(7) 19(8)

DATA> 6(9) 7(10) 6(11) 7(12) 4(13) 4(14) 5(15) 3(16) 3(17)

DATA> end
```

- **b.** From the frequency distribution (or from the histogram), the number of authors who published at least 5 papers is 33+28+19+...+5+3+3=144, so the proportion who published 5 or more papers is 144/1309 = .11, or 11%. Similarly, by adding frequencies and dividing by n = 1309, the proportion who published 10 or more papers is 39/1309 = .0298, or about 3%. The proportion who published more than 10 papers (i.e., 11 or more) is 32/1309 = .0245, or about 2.5%.
- c. No. Strictly speaking, the class described by '≥15' has no upper boundary, so it is

impossible to draw a rectangle above it having finite area (i.e., frequency).

d. The category 15-17 does have a finite width of 2, so the cumulated frequency of 11 can be plotted as a rectangle of height 6.5 over this interval. The basic rule is to make the area of the bar equal to the class frequency, so area = 11 = (width)(height) = 2(height) yields a height of 6.5.