```
> # Here I go over the caluclations for the Hopf studied in class
> restart;
> interface(showassumed=0);
```

$$1 \quad (1)$$

```
> with(LinearAlgebra):
> assume(c,'real',d,'real',r,'real',omega0,'real');
> p:=Matrix([[1/(2*c*d)],[-I/(2*omega0*(c+d))]]);
```

$$p := \begin{bmatrix} \frac{1}{2\,c\,d} \\ -\frac{\frac{1}{2}\,\mathrm{I}}{\omega 0\,(c+d)} \end{bmatrix} \quad (2)$$

```
> q:=Matrix([[c*d],[-I*omega0*(c+d)]]);
```

$$q := \begin{bmatrix} c\,d \\ -\mathrm{I}\,\omega 0\,(c+d) \end{bmatrix} \quad (3)$$

```
> B:=(x,y)->Matrix([[-r*d*x(1)*y(1)/(c+d)-c*x(1)*y(2)],[(c-d)*x(1)*
  y(2)]]);
```

$$B := (x, y) \rightarrow \mathit{Matrix}\left(\left[\left[-\frac{r\,d\,x(1)\,y(1)}{c+d} - c\,x(1)\,y(2)\right], \left[(c-d)\,x(1)\,y(2)\right]\right]\right) \quad (4)$$

```
> C:=(x,y,z)->Matrix([[-r*x(1)*y(1)*z(1)],[0]]);
```

$$C := (x, y, z) \rightarrow \mathit{Matrix}([[-r\,x(1)\,y(1)\,z(1)], [0]]) \quad (5)$$

```
> B(q,q);
```

$$\begin{bmatrix} -\frac{r\,d^3\,c^2}{c+d} + \mathrm{I}\,c^2\,d\,\omega 0\,(c+d) \\ -\mathrm{I}\,(c-d)\,c\,d\,\omega 0\,(c+d) \end{bmatrix} \quad (6)$$

```
> conjugate(q);
```

$$\begin{bmatrix} c\,d \\ \mathrm{I}\,\omega 0\,(c+d) \end{bmatrix} \quad (7)$$

```
> g20:=2*DotProduct(p,B(q,q));
```

$$g20 := \frac{-\frac{r\,d^3\,c^2}{c+d} + \mathrm{I}\,c^2\,d\,\omega 0\,(c+d)}{c\,d} + (c-d)\,c\,d \quad (8)$$

```
> g11:=DotProduct(p,B(q,conjugate(q))+B(conjugate(q),q));
```

$$g11 := -\frac{c\,d^2\,r}{c+d} \quad (9)$$

```
> g21:=2*DotProduct(p,C(q,q,conjugate(q))+C(q,conjugate(q),q)+C
  (conjugate(q),q,q));
```

$$g21 := -3\,c^2\,d^2\,r \quad (10)$$

```
> #Here is the Real Part of C1
> 1/(2*omega0)*Re(I/omega0*g20*g11+g21);
```

$$-\frac{c^2 d^2 r}{\omega 0} \quad (11)$$

```
> restart;
> eq1:=diff(x1(t),t)=r*x1(t)*(1-x1(t))-c*x1(t)*x2(t)/(alpha+x1(t));
```

$$eq1 := \frac{d}{dt} x1(t) = r\, x1(t)\, (1 - x1(t)) - \frac{c\, x1(t)\, x2(t)}{\alpha + x1(t)} \quad (12)$$

```
> eq2:=diff(x2(t),t)=-d*x2(t)+c*x1(t)*x2(t)/(alpha+x1(t));
```

$$eq2 := \frac{d}{dt} x2(t) = -d\, x2(t) + \frac{c\, x1(t)\, x2(t)}{\alpha + x1(t)} \quad (13)$$

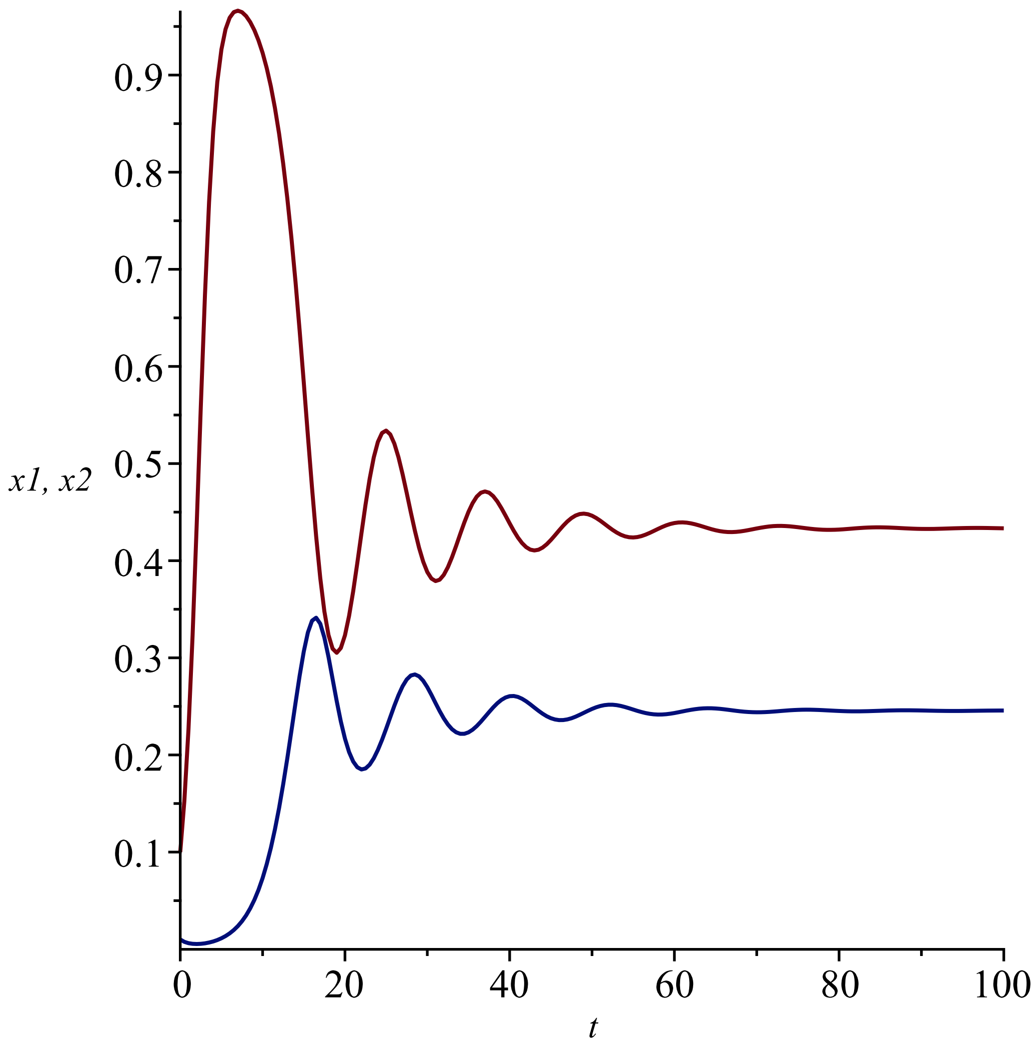
```
> r:=1:c:=2:d:=1:
> alpha:=(c-d)/(c+d)+.1;
```

$$\alpha := 0.4333333333 \quad (14)$$

```
> sol1:=dsolve({eq1,eq2,x1(0)=.1,x2(0)=.01},{x1(t),x2(t)},numeric);
```

sol1 := **proc**(*x_rkf45*) ... **end proc** (15)

```
> with(plots):
> odeplot(sol1,[[t,x1(t)],[t,x2(t)]],t=0..100);
```




```
> alpha:=(c-d)/(c+d)-.1;
```

$$\alpha := 0.2333333333 \quad (16)$$

```
> sol2:=dsolve({eq1,eq2,x1(0)=.1,x2(0)=.01},{x1(t),x2(t)},numeric);
```

$$sol2 := \mathbf{proc}(x\_rkf45) \; \ldots \; \mathbf{end\ proc} \tag{17}$$

```
> odeplot(sol2,[[t,x1(t)],[t,x2(t)]],t=0..100);
```