Math 4020/5020 - Analytic Functions
Homework #1 Due Jan 23

1. Determine the different values of the following logarithms: $\log(1)$ $\log(-1/2)$ $\log(-1+i)$

   $\log\left(\frac{a-ib}{a+ib}\right)$

2. Determine all the roots of,

   $\sin(z) = i$ $\cos(z) = 2$ $\cot(z) = 1 + i$
   You may want to use $\sin(z) = \frac{e^{iz}-e^{-iz}}{2i} \ldots$ to evaluate the functions.

3. Assume $w = f(z) = u(x,y) + iv(x,y)$ is analytic in some domain D. Show the sets of curves $u(x,y) = c_1$ and $v(x,y) = c_2$ intersect orthogonally.

4. Show the mapping $w = z^{\alpha+i\beta}$ maps the rays $\arg(z) = c_1$ and the circles $|z| = c_2$ into mutually orthogonal logarithmic spirals.

5. Use residues to calculate

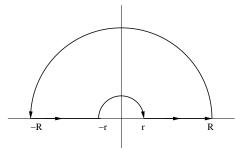$$\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+1)(x^2+2x+2)}$$

6. Show that

$$\int_0^{\infty} \frac{dx}{\sqrt{x}(x^2+1)} = \frac{\pi}{\sqrt{2}}$$

   by integrating an appropriate branch of the multi-valued function

$$f(z) = \frac{e^{(-1/2)\ln(z)}}{z^2+1}$$

   over the contour below as $R \to \infty$ and $r \to 0$.

For students of 4020, question 4 is a bonus worth an extra 5%.

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