Math 4020/5020 - Analytic Functions
Homework #2 Due Feb 11

1. Find the number of zeros of the polynomial f(z) in the first quadrant, where
   (a) $f(z) = z^3 - 3z + 6$
   (b) $f(z) = z^9 + 5z^2 + 3$
2. Find the number of zeros of $f(z)$ in the given annulus:
   (a) $f(z) = z^4 - 2z - 2$ in $\frac{1}{2} < |z| < \frac{3}{2}$,
   (b) $f(z) = ze^z - \frac{1}{4}$ in $0 < |z| < 2$.
3. Let f and g be analytic inside a simple closed curve γ and suppose that $f(z) \neq 0$ inside of γ. Show that if $|f(z)| \geq |g(z)|$ for all $z \in \gamma$ then $|f(z)| \geq |g(z)|$ for all z inside of γ. Give an example to show that the assumption that $f(z) \neq 0$ inside of γ is necessary. (Note: you will have to consider what can happen if $f(z) = 0$ on γ.)
4. Find a linear fractional transform that maps:
   (a) the circle $|z| = 1$ onto the line $Re((1 + i)w) = 0$.
   (b) the circle $|z| = 1$ onto the circle $|w - 1| = 1$.
   (c) the real axis onto the line $Re(w) = 1/2$.
5. Find the image of the following sets under $w = \frac{z-i}{z+i}$:
   (a) the real axis.
   (b) the circle $|z| = 1$.
   (c) the imaginary axis.
6. (a) Find a conformal mapping of the region $D = \{z : |z - 1| < \sqrt{2}, |z + 1| < \sqrt{2}\}$ onto the open first quadrant.
   (b) Find a conformal mapping of D onto the upper half plane. This can't be a linear fractional mapping. Why?
7. Find a conformal mapping of the quarter circle $D = \{z = x + iy : |z| < 1, x > 0, y > 0\}$ onto the upper half plane. This can't be a linear fractional mapping either. Why?

For students of 4020, question 3 is a bonus worth an extra 5%.