

Math 4020/5020 - Analytic Functions

Homework #3 Due Wednesday March 4

1. An arc of θ_0 radians in a circle is kept at temperature T_1 while the remainder of the circle is kept at T_2 . By mapping into the upper half plane, find the temperature distribution inside the circle.
2. Let D be the quartercircle $\{z : |z| < 1, x > 0, y > 0\}$. Find the electrostatic potential ϕ in D (harmonic in D) with the following boundary conditions: $\phi = 0$ on the real axis, $\phi = 1$ on the imaginary axis and $\frac{\partial \phi}{\partial n} = 0$ on the circular part (no flux).
3. Find a stream function and velocity potential function for flows with the following velocity fields:
 - (a) $f = \bar{z}$.
 - (b) $f = \sin(\bar{z})$.
 - (c) $f = \frac{\bar{z}-1}{\bar{z}+1}$

Note: If $f = u + iv$ the velocity of the flow will be given by the vector (u, v) .

4. Using the fact that the Joukowski mapping $w = z + \frac{1}{z}$ maps circles $|z| = r > 1$ to ellipses, find the complex velocity potential for flow past an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$ with no circulation.
5. Let $w = f(z)$ be a non-constant analytic function mapping a domain D to the domain E . Suppose that ψ is a smooth function on E and that ϕ is defined on D by $\phi(z) = \psi(f(z))$. Show that $\phi_{xx} + \phi_{yy} = |f'(z)|^2 (\psi_{uu} + \psi_{vv})$.
6. Find the electrostatic potential between the two cylinders with cross-sections $\{z : |z - 2| = 1\}$ and $\{z : |z + 2| = 1\}$ if the first cylinder has charge Q_1 and the second Q_2 . This question is a bonus for those enrolled in math4040.