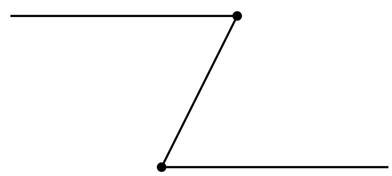
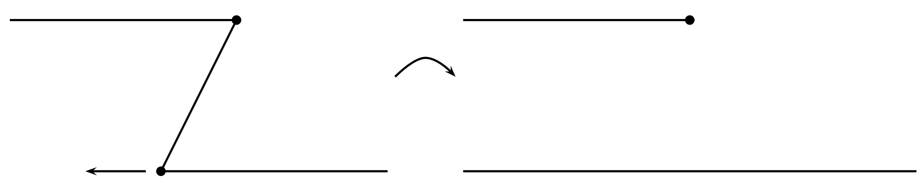
Math 4020/5020 - Analytic Functions
Homework #4 Due March 27th

1. Find the Schwartz-Christoffel transformation which maps the upper half-plane onto the triangle with vertices $\{0, i, 1\}$. Choose $x_1 = -1$, $x_2 = 1$.
2. Find a Schwartz-Christoffel transformation which maps the upper half-plane onto the domain $D = \{z : 0 < Arg(z) < 4\pi/3\}$
3. (a) Use Schwarz-Christoffel to find a mapping of the upper-half plane into the domain above the "z"-type boundary as shown below. Note: only write $f'(z)$ down, you don't need to solve for f.



   (b) Take the limit as indicated in the figure below. Find $f(z)$ explicitly in this case.



   (c) Use a computer to sketch a few streamlines for the domain in part 3b
4. Apply the ideas of Step 1 of the Riemann mapping theorem to devise invertible, conformal mappings of the following simply connected domains to domains that lie within the unit disc $|w| < 1$ and include the point $w = 0$.
   (a) The strip $0 < \Re(z) < 100$.
   (b) The double cut plane obtained by deleting the rays $(-\infty, -1]$ and $[-1, \infty)$ from $\mathbb{C}$.
5. Show that the mapping
$$z = w\frac{\lambda - w}{1 - \lambda w},$$
from $w \to z$ is a contraction if $\lambda = \frac{2\sqrt{r}}{1+r} < 1$, whenever $|w| < 1$. This question is a bonus students in 4020.