Math 4220/5220 -Introduction to PDE's
Homework #1 Due Jan. 30^{th}

1. Find the most general solution to the following PDEs:
   (a) $au_x + bu_y + cu = 0$ where a, b and c are constants.
   (b) $u_x + u_y + u = e^{x+2y}$ with $u(x,0) = 0$.
   (c) $u_x + 2u_y + (2x - y)u = 2x^2 + 3xy - 2y^2$.
2. Consider the equation $3u_y + u_{xy} = 0$.
   (a) What is its type?
   (b) Find the general solution. (Hint: Substitute $v = u_y$.)
   (c) With the auxiliary conditions $u(x,0) = e^{-3x}$ and $u_y(x,0) = 0$, does a solution exist? Is it unique?
3. The PDE for a 3-dimensional radially symmetric wave is given by,

$$u_{tt} = c^2 \left(u_{rr} + \frac{2}{r} u_r \right) .$$

   (a) Solve $u_{tt} = c^2 u_{xx}$, $u(x,0) = e^x$, $u_t(x,0) = \sin(x)$.
   (b) Change variables $v = ru$ the get the equation for v: $v_{tt} = c^2 v_{rr}$
   (c) Use this change of variables to solve the spherically symmetric wave equation in 3-dimensions with the initial conditions $u(r,0) = \phi(r)$, $u_t(r,0) = \psi(r)$ where ϕ and ψ are even functions of r.
4. For each of the following partial differential equations, identify the equation as parabolic, elliptic or hyperbolic and find a transform to put the system in a standard form.
   (a) $2u_{xx} + 4u_{xt} + 2u_{tt} = 0$
   (b) $8u_{xx} + 10u_{xy} + 2u_{yy} = 0$
   (c) $10u_{xx} + 12u_{xy} + 4u_{yy} = 0$
5. If $u(x,t)$ satisfies $u_{tt} = u_{xx}$, prove the identity

$$u(x+h, t+k) + u(x-h, t-k) = u(x+k, t+h) + u(x-k, t-h)$$

   for all x, t, h and k. Sketch the quadrilateral Q whose vertices are the arguments of the identity.

For students enrolled in Math 5220 all questions are of equal value and required.

For students enrolled in Math 4220, question 5 is a bonus question worth up to an additional 5 %.