Math 4220/5220 -Introduction to PDE's

Homework #4 Numerical Methods Due March 27

1. (a) Solve the problem $u_t = u_{xx}$ in the interval $[0, 4]$ with $u = 0$ at both ends and $u(x.0) = x(4 - x)$, using the forward difference scheme with $\Delta x = 1$ and $\Delta t = 0.25$. Calculate 4 time steps (up to $t = 1$).

   (b) Do the same with $\Delta x = 0.5$ and $\Delta t = 0.0625 = \frac{1}{16}$. Calculate 4 time steps (up to $t = 0.25$).

   (c) Compare your answers. How close are they at $x = 2.0$, $t = 0.25$?

2. Consider the equation $u_t = au_{xx} + bu$, where a and b are constants with $a > 0$. Use forward differences for u_t and centered differences for u_{xx}.

   (a) Write down the scheme with $s = \frac{\Delta t}{(\Delta x)^2}$.

   (b) Find the condition on s for stability.

3. Find an approximate solutions for $u_{tt} = u_{xx}$ on the interval $0 \leq x \leq 1$ with $u = 0$ at both ends and $u(x, 0) = \sin \pi x$ and $u_t(x, 0) = 0$. Show that the solution is periodic. Compare your answer to the exact solution. What is the period?

4. Consider the first-order equation $u_t + au_x = 0$.

   (a) Solve it exactly with the initial condition $u(x.0) = \phi(x)$.

   (b) Write down the finite difference scheme which uses the forward difference for u_t and the centered difference for u_x.

   (c) For which values of Δt and Δx is the scheme stable.

5. (a) Write down the scheme using centered differences for the equation $u_{xx} + u_{yy} = f(x, y)$.

   (b) Use it with $\Delta x = \Delta y = 0.5$ to solve the problem $u_{xx} + u_{yy} = 1$ in the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ with $u = 0$ on the boundary.

   (c) Repeat with $\Delta x = \Delta y = \frac{1}{3}$.

   (d) Compute the exact value at the center of the square and compare with your answer to part (b).

For students enrolled in Math 5220 all questions are of equal value and required.

For students enrolled in Math 4220, question 4 is a bonus question worth up to an additional 5 %.

Feel free to use my code from the course home page. You may use Matlab on the computers in the Chase basement. As well it is possible to use Octave at home. Check out

`http://www.gnu.org/software/octave/`

to download for your own platform.