# MATH/CSCI 2113
Solutions to Assigment 1

1. Pamela has 15 different books. In how many ways can she place her books on two shelves so that there is at least one book on each shelf?

   There are 15! different permutations of the books, and 14 ways to divide any permutation into two parts, where the first part goes onto the top shelf, and the second part onto the bottom shelf. So in total there are $14 \cdot 15! = 18307441152000$ ways to arrange the books. (Note: without the restriction that at least one book must be placed on each shelf, there would be 16 ways to divide the books into two parts.)

2. (a) In how many ways can the letters in the word VISITING be arranged?
   (b) For the arrangements of part (a), how many have all three I's together?

   (a) $\frac{8!}{3!} = 6720$. (b) Treat the three I's together as one letter. So $6! = 720$ arrangements where all I's are together.

3. In a social dance class, there are 12 women and 8 men. In how many ways can the 8 men be paired with 8 of the 12 women?

   $P(12, 8) = 12 \cdot 11 \cdots 5 = 19958400$. Note that order matters, since man 1 dances with woman 1 and man 2 dances with woman 2 is different from man 1 dances with woman 2 and man 2 dances with woman 1.

4. How many 4 digit integers:

   (a) contain all different digits?

      $9 \cdot 9 \cdot 8 \cdot 7 = 4536$.

   (b) do not contain the digit 5?

      $8 \cdot 9 \cdot 9 \cdot 9 = 5832$.

   (c) do not end with 5 or start with 1?

      $8 \cdot 10 \cdot 10 \cdot 9 = 7200$

   (d) do not contain more than one 5?

No five: see (b). Exactly one 5 in first position: $1 \cdot 9 \cdot 9 \cdot 9$. One five in second, third or fourth position: $8 \cdot 1 \cdot 9 \cdot 9$. Total:

$5832 + 729 + 3 \cdot 648 = 8505.$

(e) have their digits in strictly increasing order? (So the first digit is smaller than the second digit, etc. For example, 1356 and 2478 have their digits in increasing order.)

Note first that the integer cannot contain a zero. Choose four different digits between 1 and 9. This can be done in $C(9,4) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$ ways. Once the digits are chosen, their order is already determined, so there is exactly one integer with increasing digits that contains exactly these four digits. So there are 126 such integers.

5. Determine how many ways 20 coins can be selected from four large containers, one filled with pennies, one with nickels, one with dimes and one with quarters.

Distribute the 20 coins over four categories. Equivalent to the number of ways to write 20 crosses and 3 dividors. $C(23,3) = \frac{23 \cdot 22 \cdot 21}{3 \cdot 2 \cdot 1} = 1771$.

6. In the following program, $i, j, k$ and *counter* are integer variables. Determine the value that the variable *counter* will have after the segment is executed.

```
counter := 10
for i := 1 to 15 do
   for j := i to 15 do
      for k := j to 15 do
         counter := counter + 1
```

The number of iterations equals the number of triples of integers $i, j, k$, where $1 \leq i \leq j \leq k \leq 15$. This number is $C(17,3)$ (number of ways to write 14 crosses, for the 14 integers greater than 1 and less than or equal to 15, and three dividors, for $i, j, k$. $C(17,3) = \frac{17 \cdot 16 \cdot 15}{3 \cdot 2 \cdot 1} = 680$. So *counter* is increased by one 680 times. So at the end of the segment, *counter* $= 690$.