MATH/CSCI 2113

Solutions to assigment 2

1. Find the successors of the following permutations in a lexicographic ordering: (a) 15432 (b) 54312 (c) 35421.

   (a) 21345 (b) 54321 (c) 41235

2. (a) Generate all rearrangements of the word BOOST, in lexicographic order (where letters that come earlier in the alphabet are smallest).

   Hint: you can use the same method as for the generation of permutations in lex order.

1.BOOST	6.BSOTO	11.OBOST	16.OBTSO	
2.BOOTS	7.BSTOO	12.OBOTS	17.OOBST	
3.BOTOS	8.BTOOS	13.OBSOT	18.OOBTS	
4.BOTSO	9.BTOSO	14.OBSTO	19.OOSBT	
5.BSOOT	10.BTSOO	15.OBTOS	20.OOSTB	etc...

   (b) Check that the number of rearrangements you found coincides with the formula for the number of rearrangements as learned last week.

$$\frac{5!}{1!2!1!1!} = 60$$

3. (a) Determine the number of nonnegative integer solutions to the pair of equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ x_1 + x_2 + x_3 + x_4 + x_5 &= 15 \end{aligned}$$

   There are $C(6+3-1,6)$ solutions to the first equation, and $C(9+2-1,9)$ solutions to the equation $x_4+x_5 = 9$. Total:$C(6+3-1,6)C(9+2-1,9) =$ 280 solutions.

   (b) How many of these solutions have $x_3 \neq 3$? (*Hint: count by complement.*)

   Solutions with $x_3 = 3$: $C(3+2-1,3)C(9+2-1,6) = 40$.
   With $x_3 \neq 3$: $280 - 40 = 240$.

4. Describe an algorithm which, on input of two integers k and n, where $1 \leq k \leq n$, generates all subsets of size k of the set $\{1, 2, 3, \ldots, n\}$. Give a precise, step-by-step description of your algorithm. You may also use pseudocode.

procedure *combination(r,n)*
  for $i := 1$ **to** r **do** $s_i := i$
  print $s_1 s_2 \ldots s_n$
  for $i := 2$ **to** $C(n, r)$ **do**
  begin
    $m := r$
    $max_{val} := n$
    while $s_m = max_{val}$ **do**
    begin
      $m := m - 1$
      $max_{val} := max_{val} - 1$
    end
    $s_m := s_m + 1$
    for $j := m + 1$ **to** r **do** $s_j := s_{j-1} + 1$
    print $s_1 s_2 \ldots s_n$
  end
end *combination*

5. Determine how many integer solutions there are to the equation

$$x_1 + x_2 + x_3 + x_4 = 19,$$

if

(a) $0 \leq x_i < 8$ for $1 \leq i \leq 4$,

$C(19 + 4 - 1, 3) - 4C(11 + 4 - 1, 3) + 6C(3 + 4 - 1, 3) - 0 + 0$

(b) $0 \leq x_1 \leq 5$, $0 \leq x_2 \leq 6$, $3 \leq x_3 \leq 7$, $3 \leq x_4 \leq 8$.
This is equivalent to finding all solutions of $x_1 + x_2 + y_3 + y_4 = 13$, where $0 \leq x_1 \leq 5$, $0 \leq x_2 \leq 6$, $0 \leq y_3 \leq 4$, $0 \leq y_4 \leq 5$ (take $y_3 = x_3 - 3$ and $y_4 = x_4 - 3$).
$C(16, 3) - C(9, 3) - 2C(10, 3) - C(11, 3) + 2C(3, 3) + 2C(4, 3) + 2C(5, 3) - 0 + 0$