MATH 3330 — Applied Graph Theory

Assignment 5

Due Tuesday, February 13, 2007 (before class)

1. Do problem 4.3.7 of the text (find a shortest-path tree for the given graph). Show your work.

2. Show that, when the Dijkstra algorithm is applied to a graph where all edges have the same weight, the result is a BFS tree.

3. Problem 4.2.15. Characterize the graphs for which the BFS and DFS trees are identical, no matter what the "tie-breaking" priority or the starting vertex are.

4. (Variation of problem 4.3.9) Similarly to the problem from assignment 3, where MST was applied to finding the most reliable spanning tree, we can use the shortest path principle to find the most reliable connection in a network to a given vertex. Precisely, given a graph (network), and failure probabilities p_{ij}, find the most reliable path between two given nodes s and t.

   Show how to modify Dijkstra's algorithm to solve this problem (give details and justification). Illustrate your method on the graph given in 4.3.9.

5. Find a dfs-tree of the graph shown in 2.4.9. Compute the df numbers and "low" numbers (see class notes and Section 4.4) for each vertex, and use this to find all cut vertices of the graph.

6. True or false: the diameter of a graph is the maximum depth of a dfs tree of the graph.

   If true, give an argument why this is so. If false, give a counter example: an example of a graph and a vertex where the eccentricity and depth of bfs tree are not the same.

7. Follow the link given on the course Web page labelled "wire routing". The given applet finds a path from a start square to finish square in a grid with blockages. The graph it represents is as follows: the squares are its vertices, and edges correspond to adjacent squares. The applet uses one of the tree-growing algorithms discussed in class. Which one? Explain your answer.