MATH 3330 — Applied Graph Theory
Assignment 8
Due Thursday, March 29, 2007 (before class)

1. Suppose a tourist wants to travel from LA to New York city, in stages. In order to plan the most touristy route, a travel agent has identified a large number of cities that could be used as stopping places along the way, and assessed a number of routes to travel from city to city, with an assessment of "benefit" (scale 1-10) assigned to each route. For example: LA-Las Vegas, benefit 8. LA-San Fransisco, benefit 7. San Fransisco-Las Vegas, benefit 5, etc. The aim of the travel agent is to find a route from LA to New York that will maximize the benefit for the tourist.

   (a) Formulate this problem as a shortest path problem.

   (b) Why can Dijkstra's algorithm not be used to solve this problem? Explain.

   (c) Which algorithm can be used? Describe the algorithm.

   (d) What happens if the graph formed in (a) contains a directed cycle?

2. Solve the following shortest path problem with the Floyd-Warshall algorithm. The table below represents the weights of the edges. If the entry is $-$, no edge exists. Using the matrix E, find the shortest (cheapest) path from v_1 to v_3.

	v_1	v_2	v_3	v_4
v_1	-	2	3	-
v_2	3	-	-3	4
v_3	-	5	-	3
v_4	1	1	1	-

3. Describe the problem of finding a maximum weight matching (find a matching so that the sum of the weights of the edges in the matching is maximized) as a minimum cost flow problem. Carefully describe the transformation.

4. Determine the maximum flow from s to t of minimum cost in the fol-

lowing network. *Show your work.*

5. Suppose cars have to be transported from 3 manufacturing plants w_1, w_2 and w_3 to dealerships s_1, s_2, and s_3. The number of cars available at each plant is 900, 1500 and 700, respectively. The number of cars required at each dealership is 500, 1300 and 700. The cost of shipping one car from plant to dealership is given in the table below. The problem is to find the cheapest way of shipping cars from manufacturers to dealers, so that as much of the demand for cars is satisfied (however, you may not deliver more cars than the dealership requested, because of storage problems). Describe a way to solve this problem using minimum cost maximum flow.