MATH 3330 — Applied Graph Theory

Assignment 2

Due Tuesday, January 23, 2007 (before class)

(2.1.2) Find all possible isomorphism types of a connected graph with four vertices.
There are 6 different types: 2 with 3 edges (path and star), 2 with 4 edges (cycle and triangle with edge attached), 1 with 5 edges (complete minus one edge), 1 with 6 edges (complete).2 points

(2.1.13) Find all possible isomorphism types of a simple graph with three vertices and three edges.
I did not copy this question right from the book. The answer to the question above is: exactly one type, the triangle. In the book, it does not say "simple", in this case there are 13 types: 1.triangle, 2.path with one double edge, 3.path with one loop on the middle vertex, 4.path with loop on the end vertex, 5.isolated vertex and triple edge, 6.vertex with loop and separate double edge, 7.vertex with double loop and separate single edge, 8.isolated vertex and double edge with loop at one endpoint, 9.isolated vertex and single edge and loop at each endpoint, 10.isolated vertex and single edge with double loop at one endpoint. Three disconnected vertices with: 11.one loop at each vertex, double loop at one vertex, 12.single loop at another, 13.triple loop at one vertex. 1 point, both answers were counted as correct.

(2.5.6) Determine whether the given graphs are isomorphic. *No. One graph has diameter 4, the other 5. 2points*

(2.5.11) Determine whether the given graphs are isomorphic. *No. Degree sequences are different. 1 point.*

(2.5.25) Determine whether the given digraphs are isomorphic. *Yes. Isomorphism:* $f(1) = b$, $f(2) = f$, $f(3) = d$, $f(4) = e$, $f(5) = a$, $f(6) = c$. *2 points.*

(2.8.26) Find the diameter of the cartesian product $C_m \times C_n$ of two cycle graphs.
Note first that C_m *has diameter* $\lfloor m/2 \rfloor$ *and* C_n *has diameter* $\lfloor n/2 \rfloor$. *Then* $C_m \times C_n$ *has diameter* $\lfloor m/2 \rfloor + \lfloor n/2 \rfloor$. *Namely, consider a path from a vertex* (u, v) *to* (u', v') *in* $C_m \times C_n$. *Now, if we write down the first coordinates of each pair on this path, and eliminate successive*

duplications, then we get a path from u to u' in C_m. If we do the same with the second coordinates, we get a path from v to v' in C_m. So each step in the path either corresponds to a step of a path C_m from u to u', of the form $(x, y) - (x', y)$, where $\{x, x'\}$ is an edge of C_m, or it corresponds to a step of a path C_n from v to v', of the form $(x, y) - (x, y')$, where $\{y, y'\}$ is an edge of C_n. So the length of this path is the sum of the length of a path from u to u' in C_m, and a path from v to v' in C_n. Therefore, the length of a longest path in $C_m \times C_n$ equals the sum of the lengths of longest paths in C_m and C_n. Note that this argument holds for all graphs, not just for cycles. So, in general, $diam(G \times H) = diam(G) + diam(H)$. 2 points.