MATH 3330 — Applied Graph Theory
Assignment 7 – Solutions

1. Find a maximum flow and minimum cut in the networks of problems 13.1.2 and 13.1.4. Use trial and error.

   (13.1.2) Maximum flow has value 12. Below the amount of flow in each edge is given.

su	sx	uw	uv	wz	xv	xz	vy	yz	yt	zt
7	5	4	3	4	3	2	6	0	6	6

   The minimum cut is $< V_s, V_t >$ *where* $V_s = \{s, x\}$, $V_t = \{u, v, w, y, z, t\}$. *Capacity equals 12.*

   (13.1.4) Max flow has value 18. Minimum cut given by $V_s = \{s, x\}$. *2 points. Many possibilities to make the flow, but notice that all edges in the cut must be filled to capacity. 2 points*

2. Suppose a capacitated s-t network N is given. Suppose that the directed version of depth-first search is applied with root s. Let $A \subseteq V(N)$ be the vertices that are part of the dfs tree. Assume that $t \notin A$.

   What is the maximum value of an s-t flow in this network? Justify your answer by giving a minimum cut.

   The maximum flow is zero (zero flow in all edges). A minimum cut is $< A, V(N) - A >$. *Since no node in* $V(N) - A$ *can be reached from* s, *there are no edges with head in* $V(N) - A$, *tail in* A. *So the capacity of the cut equals zero. 1 point.*

3. Text, problem 13.1.10.

   Add a source s *and arcs* ss_1 *and* ss_2, *and a new sink* t, *and arcs* t_1t *and* t_2t, *all with capacity infinity. Max flow has value 14, min cut:* $V_s = \{s, s_1, s_2, u, z\}$. *2 points*

4. Text, problem 13.2.2. Find a maximum flow and minimum cut by applying the max-flow algorithm.

   Full points only if you show your work, including the auxiliary graph in each step. 2 points.

5. Text, problem 13.2.7. *Use max flow, min cut. I assumed the capacities indicated in the network represented hundreds.*

   Add a source s*, arc* sX *with capacity 5,* sY *with capacity 5, and* sZ *with capacity 9 (supply, in hundreds). Add sink* t*, arc* At *with capacity 7,* Bt *with capacity 6, and arc* Ct *with capacity 6. Max flow has value 16, min cut:* $V_s = \{s, Z\}$*. Translate solution back into original problem. 2 points*

6. Text, problem 13.2.9. *Form a network as follows. Vertices: one for each type of chemical, and one for each truck, and a source* s *and a sink* t*. Arcs: From* s *to each chemical vertex, with capacity 3 (number of containers available). Arcs from each chemical vertex to each truck vertex, with capacity 1. Arcs from each truck vertex to* t*, with capacity equal to the number of containers the truck can carry. Apply max flow. The flow in the arc from a chemical vertex to a truck vertex will represent the number of containers (zero or one) of that type of chemical travelling in that truck. 1 point*