Topics in Graph Theory – Problem set 10
Due Thursday, April 3, beginning of class

1. Consider the graph sequence of complete bipartite graphs: $\{K_{n,n}\}_{n=1}^{\infty}$. *Note that $K_{n,n}$ has $2n$ vertices, so the denominator of the homomorphism densities should be given in terms of $2n$, not n* (a) Find the homomorphism densities $t(F, K_{3,3})$ for F an edge K_2, a triangle K_3, and a path P_2. For each choice of F, give one homorphism explicitly. (b) Find the homomorphism densities $\lim_{n\to\infty} t(F, K_{n,n})$ for the three choices of F listed in (a). (c) Give function w_G (as defined in class on Thursday March 27) for G=$K_{5,5}$. (d) Give your guess for the function w that is the limit of this sequence, and compute $t(F, w)$ for the three choices of F. *Hint: w is a function that only takes values zero or one.*

2. Suppose G is the 5-cycle, W_5, consisting of a 5-cycle with a universal vertex. Its vertices are labelled $v_1, v_2 \ldots, v_5, v_6$. (a) Give w_G. (b) Give the expected number of edges in the w-random graph $G(n, w_G)$.

3. Given is a function w with the homomorphism density $t(K_2, w) = 0.43$. What is the expected number of edges in the w-random graph $G(n, w)$? Motivate your answer.

4. Consider the graph sequences of paths: $\{P_n\}_{n=1}^{\infty}$. (a) Give the homomorphism densities $t(F, P_n)$ for $F = K_2$, $F = K_3$ and $F = P_2$, and the limit as n goes to infinity. (b) Give your guess for the function w that is the limit of this sequence, and motivate your answer.

5. Suppose graphs G, H and F are given. (a) Let f be an isomorphism from G to H, and ν a homomorphism from F to G. Show that $f \circ \nu$ is a homomorphism from F to H. (b) Use this to show that $t(F, G) = t(F, H)$.