Topics in Graph Theory – Problem set 4

Due Tuesday, Feb. 11, beginning of class

1. Review Cuong's and Poppy's presentations of Jan 30. (a) Find a maximum matching in the circulant graph $C(13,3)$. Prove that it is a maximum matching. (b) Find a minimum edge cover in the same graph, and use Gallai's theorem to show it is a minimum edge cover. (c) Find a matching M and edge cover E in the same graph so that $|M|+|E|=n$ but M is not maximum and E is not minimum.

2. Review Melanie's presentation of Jan. 30. Let G be the circulant graph $C(16,3)$, and H be the circulant graph $C(11,2)$. Label vertices $0,1,2,\ldots$ in each. Find 4-colourings of G and H. (b) Consider the graph K formed by adding the following 4 edges: one edge between vertices labelled 0 in G and H, one edge between the vertex labelled 1 in G and the vertex labelled 2 in H, two edges between the vertex labelled 2 in G and the vertices labelled 9 and 10 in H. Use the methods from Melanie's presentation to find a 4-colouring of K.

3. Consider the following list assignment for the line graph of $K_{4,4}$. Find a list colouring, *using the method shown in class*. (Just giving a list colouring will not count). First find the desired orientation using a latin square. Make sure the out-degree of each vertex is 3. Then, in each step, draw the colour graph, and identify the kernel you use (you do not need to use Gale-Shapley, but can find it by inspection).

	y_1	y_2	y_3	y_4
x_1	1,2,3,4	1,2,4,5	1.3.4.5	2,3,4,5
x_2	2,3,4,5	2,3,4,5	1,2,3,4	1,2,3,5
x_3	1,2,3,4	2,3,4,5	1,3,4,5	1,3,4,5
x_4	2,3,4,5	1,2,4,5	1,2,4,5	1,2,3,5

4. Let G be a graph with chromatic number k and size n. Fix an integer t, $1 \leq t < k$. Show that G has an induced subgraph H of size at least $(t/k)n$ which is t-colourable. (In other words, show that at least a fraction t/k of the vertices can be coloured if only t colours are available.)

5. [MATH 4330/CSCI 4115] Show that for interval graphs, the list colouring number equals the chromatic number.

6. [MATH 5330] Let $G = (V, E)$ be a 4-choosable graph, of size n. Let L be a list assignment for G so that each list is of size 2. (a) Show that at least half of the vertices of G can be coloured with these lists. Precisely, show that there is a set $W \subset V$ so that $G[W]$ with list assignment given by L restricted to W has a list colouring, and $|W| \geq n/2$.
In general, it is an open conjecture whether, for a k-choosable graph of size n, and a list assignment with lists of size $t < k$, there always is a subgraph of size $(t/k)n$ which can be coloured with those lists. (b) Part (a) shows that the conjecture is true for $k = 4$, $t = 2$. Generalize this approach to other values of t and k. *Note: the case* $t = 2$, $k = 3$ *is still wide open!*