Some Fluctuation Results on Draw-down Times for Spectrally Negative Lévy Processes

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Abstract

Lévy processes have been applied widely in a lot of fields of science. Some examples can be listed here such as for the study of turbulence, laser cooling and quantum field theory in physics; for the study of networks, queues and dams in engineering; for continuous time-series models and risk models in economics, for risk theory in actuarial mathematics, and of course, in mathematical finance, for the stock price in the market and calculations of insurance and re-insurance risk.

One of the most obvious and fundamental problems that can be stated for Lévy processes, particularly in relation to their role as modeling tools, is the distributional characterization of the time at which a Lévy process first exits an interval together with its overshoot and undershoot beyond the boundary of that interval. With the solution of one-sided and two-sided exit problems on hand, researchers can develop lots of relative properties of Lévy processes.

In this talk, we first introduce and review some fluctuation theory of Lévy processes, especially for general spectrally negative Lévy processes and for spectrally negative Lévy taxed processes. Then we consider a more realistic model by introducing draw-down time, which is the first time a process falls below a predetermined draw-down level which is a function of the running maximum. Particularly, we present the expressions for the classical two-sided exit problems for these processes with draw-down times in terms of scale functions. We also find the expressions for the discounted present values of tax payments with draw- down time in place of ruin time. Finally, we obtain the expression of the occupation times for the general spectrally negative Lévy processes to spend in draw-down interval killed on either exiting a fix upper level or a draw-down lower level.