NUMBER THEORY SEMINAR

Dynamics of finite linear cellular automata over $\mathbb{Z}_N$

Franklin Mendivil

Acadia University

This talk is based partially on the Honours thesis of Donald Patterson.

WHEN: Wed 07 Apr 2010, 3:30 p.m.

WHERE: Chase 319

ABSTRACT:

The *Ducci game* is a classical recreational mathematics puzzle which operates on circular vectors of integers, using the rule

$$(x_1, x_2, \ldots, x_n) \mapsto (|x_2 - x_1|, |x_3 - x_2|, \ldots, |x_1 - x_n|).$$

It turns out that for any starting sequence there is an $M \geq 0$ so that eventually the iterates are from $\{0, M\}^n$. This means that we can think of the eventual dynamics as acting on $\mathbb{Z}_2$ by the linear map

$$(x_1, x_2, \ldots, x_n) \mapsto (x_2 + x_1, x_3 + x_2, \ldots, x_1 + x_n).$$

In this talk, after giving a brief pre-history of this problem, we examine the general situation of the dynamics of a linear map acting on $\mathbb{Z}_N^L$. We first discuss the special case of N being a prime, and then use the Chinese Remainder Theorem to reduce to the case of N being a prime power. We present some results which relate the dynamics modulo p^{k+1} to those modulo p^k.

Any questions, please email: rnoble@mathstat.dal.ca.