MATH 2030: ASSIGNMENT 1

Geometry and Algebra of Vectors

Q.1: pg 16, q 1,2. For each vector draw the vector in standard position and with its tail at the point $(1, -3)$:

$$\mathbf{u} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \ \mathbf{z} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}.$$

Q.2: pg 17, 16. Simplify the vector expression, and indicate which properties from Theorem 0.6 are being used.

$$-3(\mathbf{u} - \mathbf{w}) + 2(\mathbf{u} + 2\mathbf{v}) + 3(\mathbf{w} - \mathbf{v})$$

Q.3: pg 17, pg 17,18. Solve for the vector $\mathbf{w}$ in terms of $\mathbf{u}$ and $\mathbf{v}$:

- $\mathbf{w} - \mathbf{u} = 2(\mathbf{w} - 2\mathbf{u})$
- $\mathbf{w} + 2\mathbf{u} - \mathbf{v} = 3(\mathbf{w} + \mathbf{u}) - 2(2\mathbf{u} - 2\mathbf{v})$

Q.4: pg 17, p 21. In $\mathbb{R}^2$ given two vectors $\mathbf{u}$, $\mathbf{v}$ such that $\mathbf{u} \neq c\mathbf{v}$ for all $c \in \mathbb{R}$, we may fill the coordinate planes with parallelograms with $\mathbf{u}$ and $\mathbf{v}$ as sides. In essence this construction gives a new coordinate system for the plane, e.g. consider $\mathbf{u}^t = [1, 0]$ and $\mathbf{v}^t = [0, 1]$ - these produce the standard coordinate axes.
Given $\mathbf{u}^t = [1, -1]$ and $\mathbf{v}^t = [1, 1]$, draw the standard coordinate axes on the same diagram as the axes relative to $\mathbf{u}$ and $\mathbf{v}$. Use these to find $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$, where $\mathbf{w}^t = [2, 6]$.

Q.5: pg 17 q 44,46,48. Solve the given equation or indicate there is no solution

- $2x = 1$ in $\mathbb{Z}_3$
- $x + 3 = 2$ in $\mathbb{Z}_5$
- $2x = 1$ in $\mathbb{Z}_5$

The dot product

Q.6: pg 29 q 4,12. Given $\mathbf{u}^t = [1.5, 0.4, -2.1]$ and $\mathbf{v}^t = [3.0, 5.2, -0.6]$ calculate $\mathbf{u} \cdot \mathbf{v}$, $||\mathbf{u}||$ and finally give the unit vector in the direction of $\mathbf{u}$.

Q.7: pg 29 q 20,26. Determine the angle between $\mathbf{u} = [5, 4, -3]$ and $\mathbf{v} = [1, -2, -1]$ is acute, obtuse or a right angle by calculating it explicitly.

Q.8: pg 29 q 36. An airplane heading due east has a velocity of 200 miles per hour. A wind is blowing from the north at 40 miles per hour. What is the resultant velocity of the plane?

Q.9: pg 30 q 42. Find the projection of $\mathbf{v}$ onto $\mathbf{u}$ where $\mathbf{u}^t = [\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}]$ and $\mathbf{v}^t = [2, -2, 2]$.

Q.10: pg 30 q 60. Suppose we know that $\mathbf{u}\cdot\mathbf{v} = \mathbf{u}\cdot\mathbf{w}$. Does it follow that $\mathbf{v} = \mathbf{w}$. If it does, give a proof that is valid in $\mathbb{R}^n$; otherwise, give a *counterexample* (that is, a specific choice of vectors for which $\mathbf{u}\cdot\mathbf{v} = \mathbf{u}\cdot\mathbf{w}$ but $\mathbf{v} \neq \mathbf{w}$.

Lines and Planes

Q.11: pg 44 q 6. Write the equation of the line passing through P with direction vector $\mathbf{d}$ in vector form and parametric form:

$$P = (-3, 1, 2),\ \mathbf{d} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}.$$

Q.12: pg 44 q 12 . Give the vector equation of the line passing through $P = (4, -1, 3)$ and $Q = (2, 1, 3)$.

Q.13: pg 44 q 14 . Give the vector equation of the plane passing through $P = (1, 0, 0)$, $Q = (0, 1, 0)$ and $R = (0, 0, 1)$.

Q.14: pg 45 q 20 . Find the vector form of the equation of the line in $\mathbb{R}^2$ that passes through $P = (2, -1)$ and is perpendicular to the line with general equation $2x - 3y = 1$.

Q.15: pg 45 q 27 . Find the distance from the point $Q = (2, 2)$ and the line ℓ with the equation,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Q.16: pg 46 q 36 . Find the distance between the parallel lines:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Q.17: pg 46 q 43 . Find the acute angle between the planes with the equations:

$$x + y + z = 0,\ 2x + y - 2z = 0.$$

Applications of Vectors

Q.18: pg 58 q 18,20 . A parity check code vector $\mathbf{v}$ is given, determine whether a single error could have occurred in the transmission of $\mathbf{v}$:

- $\mathbf{v} = [1, 1, 1, 0, 1, 1]$
- $\mathbf{v} = [1, 1, 0, 1, 0, 1, 1, 1]$

Q.19: pg 58 q 24 . Consider the UPC $[0, 4, 6, 9, 5, 6, 1, 8, 2, 0, 1, 5]$

- Show that this UPC cannot be correct.
- Assuming that a single error was made and that the incorrect digit is the 6 in the third entry, find the correct UPC.

Q.20: pg 58 q 30 .

- Prove that if a transposition error is made in the second and third entries of the UPC with
$$[0, 7, 4, 9, 2, 7, 0, 2, 0, 9, 4, 6]$$
the error will be detected.
- Show that there is a transposition involving two adjacent entries of the UPC in the first part that would not be detected.

References

[1] D. Poole, Linear Algebra: A modern introduction - 3rd Edition, Brooks/Cole (2012).