Lifting PIE limits with strict projections *

Martin Szyld
Dalhousie University

Abstract. We give a direct proof of the lifting of any PIE limit [1] to the 2-category of algebras and pseudomorphisms [2], which specifies precisely which of the projections of the lifted limit are strict and detect strictness. In [2], several of these limits were lifted one by one, so as to keep track of these projections in each case. We work in the more general context of weak algebra morphisms [3], so as to include the case of lax morphisms of algebras [4] as well. From our result it follows that PIE limits are also all simultaneously lifted in this case, provided some specified arrows of the diagram are pseudomorphisms. Again, this unifies the previously known lifting of many particular PIE limits, which were also treated separately in [4]. During the talk I plan to give an overview of these notions and present the lifting results, which appear in [5].

It is now known [6, §6.4] that PIE limits are precisely the limits that can be lifted to all 2-categories of algebras and pseudomorphisms. However, as the authors point out themselves, the results so obtained contain no information specifying the strict and strictness-detecting projections. In op. cit. it is shown that PIE weights coincide with the coalgebras for a particular comonad, and it is this fact that is used for their lifting. Instead, in [5], the *trick* is to replace weighted limits with what we call σ-s-limits [7], [3]. These are a slight modification of conical lax limits, in which one requires that certain 2-cells of the lax cone, associated to a subcategory Σ of morphisms in the domain 2-category, are identities. The concept is due to Gray [8], and Street has shown in [9] that these have the same expressive power as weighted 2-limits. Throughout our work with σ-limits, we have highlighted the importance of these results by using these same ideas to show that "weighted σ-limits are conical", just like weighted 1-limits. Now, the condition in [1] characterizing PIE weights translates nicely to the setting of σ-limits, as asking that each connected component of Σ has an initial object. It is precisely these initial objects that provide the family of strict and strictness-detecting projections. Also, in the case of a diagram with lax morphisms as in [4], it is the arrows from an initial object into each object that should be mapped to pseudomorphisms for a PIE limit to be lifted. It is with this formulation that the proof of the fact that an individual PIE limit lifts becomes simple and elementary, and can be applied to diagrams of lax as well as pseudomorphisms.

References

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