

A Tannakian approach to Grothendieck Galois theory

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Grothendieck Galois theory

We begin with

$$\begin{array}{c} \mathcal{C} \\ \downarrow F \\ \mathcal{E}ns_{<\infty} \end{array}$$

Under certain hypotheses we have a lifting of F

$$\begin{array}{ccc} \mathcal{C} & \xrightarrow[\approx]{\tilde{F}} & \beta Aut(F)_{<\infty} \\ \downarrow F & & \swarrow U \\ \mathcal{E}ns_{<\infty} & & \end{array}$$

which is an equivalence of categories.

localic Galois theory (by Dubuc)

- We may omit the finiteness hypothesis on F .
- Now the group $Aut(F)$ doesn't yield the equivalence of categories.
- We construct instead a localic group $\ell Aut(F)$ whose points are the automorphisms of F .
- Under the finiteness hypothesis on F , $\ell Aut(F)$ is isomorphic to the group of its points.

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Tannaka theory by Joyal-Street

We begin with

$$\begin{array}{c} \mathcal{D} \\ T \downarrow \\ \text{Vec}_{<\infty} \end{array}$$

Under certain hypotheses we have a lifting of T

$$\begin{array}{ccc} \mathcal{D} & \xrightarrow[\approx]{\tilde{T}} & \text{Cmd}_{<\infty}(\text{End}^V(T)) \\ T \downarrow & \swarrow U & \\ \text{Vec}_{<\infty} & & \end{array}$$

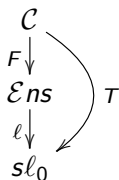
which is an equivalence of categories.

The generalization “ \mathcal{V} instead of Vec ”

- $Vec \rightsquigarrow \mathcal{V}$ an arbitrary tensor category,
 $Vec_{<\infty} \rightsquigarrow \mathcal{V}_0$ subcategory of objects that have a dual.
- The definitions and constructions can be generalized.
- $End^{\vee}(T)$ is now a Hopf algebra in \mathcal{V} .

Note: It is an open problem if \tilde{T} is an equivalence in this general case.

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Considering $\mathcal{V} = sl$ 

$$sl_0 = \{\ell X\}_{X \in \mathcal{E}ns}$$

$End^{\vee}(T)$: Hopf algebra in $sl \iff$ localic group.

Then we can compare the Galois construction $\ell Aut(F)$ with the Tannakian construction $End^{\vee}(T)$.

Main results (so far)

- We have a surjective locale morphism $\ell Aut(F) \rightarrow End^{\vee}(T)$.
- $Points(End^{\vee}(T)) = Aut(T) = Aut(F) = Points(\ell Aut(F))$.
- For the Grothendieck case, in which $\ell Aut(F)$ has enough points, so does $End^{\vee}(T)$ and therefore both constructions are isomorphic.
- In this case, since

action of the localic group \iff comodule of the Hopf
 in a set X algebra in ℓX ,

it follows that the Galois and Tannaka theorems are equivalent.

Thank you!