An Invitation to Double Categories and Double Fibrations

Martin Szyld

Joint work with Geoff Cruttwell, Michael Lambert, and Dorette Pronk

I will present our work on the notion of fibration for double categories [1]. I will not assume prior knowledge about double categories. Instead, I will present during the talk some basic aspects of the theory of double categories that were relevant to our work and that I have found appealing, hence the "invitation to double categories" title.

Double categories are (pseudo) category objects in the 2-category of categories. I will give our definition of a double fibration as a (pseudo) category object in the 2-category of fibrations. I will then show how, much like functors are category objects in an arrow category, double fibrations correspond to double functors between double categories that satisfy certain properties. We have shown that these properties are precisely the ones making the double functor an internal fibration in the 2-category of double categories.

I will also mention a "Grothendieck/Elements construction theorem" that we show in [1], which is an equivalence of categories between double fibrations and an appropriate category of "indexed double categories". This is the non-discrete case of the equivalence given in [2], where the Elements construction was that given by Bob Paré in [3]. When thinking of monoidal categories as double categories, we can also recover from our theorem the equivalence between monoidal fibrations and monoidal indexed categories as in [4, 5]. Finally, the "double Grothendieck construction" introduced in [6] can be seen as well as an instance of our construction.

References

[1] DOUBLE FIBRATIONS, in preparation (could be out there by the moment of my talk!)

[2] DISCRETE DOUBLE FIBRATIONS, MICHAEL LAMBERT, TAC 37 (2021).

[3] YONEDA THEORY FOR DOUBLE CATEGORIES, ROBERT PARE, TAC 25 (2011).

[4] FRAMED BICATEGORIES AND MONOIDAL FIBRATIONS, MICHAEL SHULMAN, TAC 20 (2008).

[5] MONOIDAL GROTHENDIECK CONSTRUCTION, JOE MOELLER AND CHRISTINA VASILAKOPOULOU, TAC 35 (2020).

[6] DAVID JAZ MYERS, DOUBLE CATEGORIES OF OPEN DYNAMICAL SYSTEMS, ELECTRONIC PROCEEDINGS IN THEORETICAL COMPUTER SCIENCE 333 (2021).