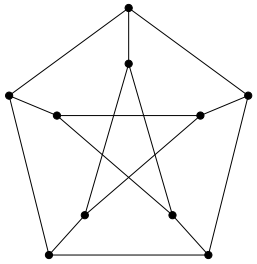
MATH 2113 - Assignment 8

Due: Mar 21

1. Let n be a large number (say $n > 100$). Describe how to construct a connected non-planar graph which has n vertices and as few edges as possible.

2. Prove that the Petersen graph (shown below) is not planar by finding a subgraph that is homeomorphic to $K_{3,3}$ or K_5.



3. (page 721) 11.5.6

4. (page 722) 11.5.30

5. Prove that the complement of the complement of G, $\overline{\overline{G}}$, is isomorphic to G.

6. Prove that the complement of a disconnected graph is connected.

7. Let G be a graph which is isomorphic to its complement $\overline{G}$. Prove that G must have $4k$ or $4k + 1$ vertices for some integer k.

8. Find (with proof!) the chromatic number of the Petersen graph.

9. Prove that if G is planar, then there must be some vertex with degree at most 5.

10. Using the result of problem 9, prove by induction on $|V| = n$ that any planar graph can be coloured with at most 6 colours.