MATH 2113 - Assignment 9

Due: Apr. 1

1. Determine if each of the following sets are rings under regular addition and multiplication:

(a) $S = \{a + \sqrt{3}b\}$
(b) $S = \{a + \sqrt{3}b + \sqrt{5}c\}$

where a, b, c are rational numbers in both cases.

2. (a) Prove $(-a) \cdot b = -(a \cdot b)$. (Part of (f), page 6.)

(b) Let $(S, \oplus, \circ)$ be a structure where $S = \mathcal{R}$, and $\oplus, \circ$ are defined by

$$\forall x, y \in S, \quad x \oplus y = x + y - 1; \quad x \circ y = x + y - xy.$$

Is this structure a ring? Explain.

3. Let $(S, +, \cdot)$ be a ring. Prove

(a) $a \cdot (b - c) = a \cdot b - (a \cdot c)$.

(b) $(b - c) \cdot a = b \cdot a - (c \cdot a)$.

4. Let $(S, +, \cdot)$ be a ring.

(a) Prove that a unit in the ring cannot also be a divisor of zero.

(b) If $a, b \in S$ are units, is $(a + b)$ a unit? Prove your claim.

5. Below are tables for a ring with elements $\{s, t, x, y\}$. Using the axioms for a ring, fill in the missing entries in the multiplication table.

Is this a commutative ring? Does it have a unity? Are there any units? Is the ring an integral domain or a field? Prove your claims.

$+$	s	t	x	y
s	s	t	x	y
t	t	s	y	x
x	x	y	s	t
y	y	x	t	s

$\cdot$	s	t	x	y
s	s	s	s	s
t	s	t	?	?
x	s	t	?	y
y	s	?	s	?

6. Let $\alpha = 2^{\frac{1}{3}}$. Prove that

$$(\{a + b\alpha + c\alpha^2\}, +, \cdot)$$

for a, b, c rational, is a ring under the usual addition and multiplication.

7. Let $R = (S, +, \cdot)$ be a ring which is not a field. Is it possible for R to have a subring which is a field? Either prove that this is not possible or give an example of a ring with divisors of zero which has a subring which is a field.

8. Let D be an integral domain. Prove that if $a^2 = 1$ then $a \ \pm 1$. Is this true in a ring also? Prove it, or give a counter example.

9. Exercise 2, Section 1.6, page 8 of the notes. (Note the definitions in exercise 1.)

10. Let S be a set, and $P(S)$ the power set of S. Prove that $(P(S), \Delta, \cap)$ is a ring, where Δ is the symmetric difference, and $\cap$ is set intersection. Is this a ring? Prove your claim.