MATH 2113 - Midterm

February 18

1. A bag of marbles contains 4 which are red, 4 which are blue and 4 which are green.

   a) How many marbles must be chosen from the bag to guarantee that three are the same colour?
   b) If you draw three marbles at random from the bag, what is the probability that they are the same colour?
   c) How many marbles must be chosen from the bag to guarantee that there is at least one of every colour?
   d) If you draw three marbles at random from the bag, what is the probability that they are all different colours?

2. For how many integers from 1 to 9999 is the sum of their digits equal to 11?

3. a) Define $P(A|B)$.
   b) Prove that if $P(A) = P(B)$ then $P(A|B) = P(B|A)$
   c) Define what it means for A and B to be independent events.

   Suppose there are three suppliers of computer parts X, Y and Z. Where 5% of X's products are superior quality, 10% of Y's products are superior quality and 15% of Z's products are superior quality. A particular store gets 50% of its parts from X, 30% from Y and 20% from Z.

   d) If a unit is purchased, what is the probability that it is superior quality?
   e) If a unit in the store is found to be superior quality, which supplier is most likely to have come from?

4. a) Define the mathematical expression "n choose r".
   b) Prove Pascal's identity:

$$\begin{pmatrix} n+1 \\ r \end{pmatrix} = \begin{pmatrix} n \\ r \end{pmatrix} + \begin{pmatrix} n \\ r-1 \end{pmatrix}$$

5. Let $D = \{1, 2, 3, ..., 52\}$ and define $s : D \to D$ by

$$s(x) = \begin{cases} 2x - 1 & \text{if } 1 \leq x \leq 26 \\ \\ 2x - 52 & \text{if } 27 \leq x \leq 52 \end{cases}$$

   a) Is s a well defined function?
   b) Is s one to one?
   c) Is s onto?
   d) Find an expression for $s \circ s$.

6. Let $D = \{1, 2, 3, ..., 52\}$ and let c be a constant such that $1 \leq c \leq 52$. Define $f_c : D \to D$ by

$$f_c(x) = \begin{cases} x - c & \text{if } x > c \\ \\ x + 52 - c & \text{if } x \leq c \end{cases}$$

   a) Prove that f_c is a well defined function.
   b) Prove that f_c is one to one for all c.
   c) Prove that f_c is onto for all c.
   d) Find an expression for f_c^{-1}.