MATH 2113 - Assignment 1 Solutions

Due: Jan 14

6.1.4 - The set of all black cards with an even number is

$$\{2\clubsuit, 4\clubsuit, 6\clubsuit, 8\clubsuit, 10\clubsuit, 2\spadesuit, 4\spadesuit, 6\spadesuit, 8\spadesuit, 10\spadesuit\}$$

There are 10 elements in the set and there are 52 in the sample space, so the probability of this event can be expressed as $P(E) = \frac{|E|}{|S|} = \frac{10}{52} = \frac{5}{26}$.

6.1.12 (b) Each of the following sets represent the associated event:
(i) $\{GBB, BGB, BBG\}$
(ii) $\{GGB, GBG, BGG, GGG\}$
(iii) $\{BBB\}$

6.1.19 (b) - The set of outcomes for this event is

$$\{B_1B_1, B_1B_2, B_1W_1, B_1W_2, B_1W_3, B_2B_1, B_2B_2, B_2W_1, B_2W_2, B_2W_3\}$$

We are told from part (a) that there are 25 outcomes in the sample space. Therefore, the probability of this event is $P(E) = \frac{|E|}{|S|} = \frac{10}{25} = \frac{2}{5}$.

6.1.19 (c) - The set of outcomes for this event is

$$\{W_1W_1, W_1W_2, W_1W_3, W_2W_1, W_2W_2, W_2W_3, W_3W_1, W_3W_2, W_3W_3\}$$

We are told from part (a) that there are 25 outcomes in the sample space. Therefore, the probability of this event is $P(E) = \frac{|E|}{|S|} = \frac{9}{25}$.

6.1.22 - The three digit integers are 100 through 999. The first multiple of 6 is 102 while the last one is 996. We can rewrite these numbers as 102 = 96 +(1)(6) and 996 = 96 + (150)(6). Therefore, we have exactly 150 3-digit numbers which are multiples of 6. There are 900 3-digit numbers, so the probability that a randomly chosen 3-number is a multiple of 6 is $\frac{150}{900} = \frac{1}{6}$.

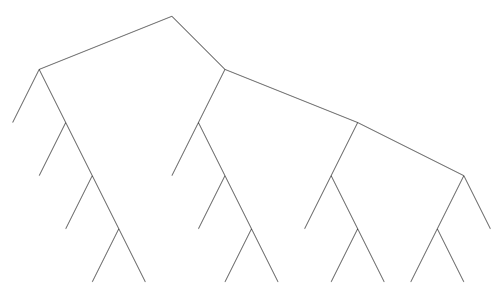
6.1.29 - Using Theorem 6.1.1, we have that $n - m + 1 = 87$ and $n = 326$. Substituting, we find that $m = 326 - 87 + 1 = 240$.

6.1.32 - Every 7th day is a Sunday, so we can simply divide to find that there are $\lfloor \frac{365}{7} \rfloor = 52$ Sundays in the year. To count the number of Mondays, we must first note that a Monday occurs every 7th day starting from January 2nd. Adding the one on January first gives us a total of $1 + \lfloor \frac{364}{7} \rfloor = 53$ Mondays in that year.

6.1.33 - We begin by fixing a particular m and considering the base case for which $n = m$. Clearly, there is only one element in our list and $n - m + 1 = m - m + 1 = 1$. Therefore, it is true in the base case.

Now assume that the statement holds for the case when $n = k$.That is to say that there are $k - m + 1$ numbers from m to k inclusive. When $n = k + 1$ we consider the numbers from m to $k + 1$. This is exactly the numbers $m, ..., k, k + 1$. By the induction hypothesis, we know that from m to k there are $k - m + 1$. So, with one more number added there are $(k - m + 1) + 1 = (k + 1) - m + 1$ as desired. Therefore, the statement is true for $n = k + 1$. Since m can be any arbitrary integer, by mathematical induction the statement is always true.

6.2.2 - The tree would look as follows:



In this diagram, the top of the tree represents A having already won two games. Each time A wins a game we take the left branch, and when B wins, we take the right branch. Each leaf of the tree represents an outcome.

6.2.15 - The three numbers may be chosen independently and each choice has 30 options. Therefore, using the multiplication rule, there are (30)(30)(30) = 27000 different combinations possible.

If we don't allow repeated numbers, we only have 29 choices for the second number and 28 for the third. Therefore, there are $(30)(29)(28) = 24360$ combinations without repeated numbers.

6.2.16 - We can use the multiplication rule for all three parts. In (a) we find $(4)(3)(4)(4) = 192$ different PINs. For (b), we get $(4)(1)(4)(3) = 48$. Finally, for (c) we note that we have 10, 9,8,7 choices respectively for the 1st, 2nd, 3rd and 4th digits of the PIN. Therefore, there are $(10)(9)(8)(7) = 5040$ different numeric sequences with no repeated digit.