MATH 2113 - Assignment 5 Solutions

Due: Feb 16

6.2.20 - a) Using theorem 6.1.1, the number of elements in a list from n to m is $m - n + 1$. Therefore, from 1000 to 9999 there are 9999-1000+1 = 9000 elements.

b) Since even and odd numbers alternate, half of our list is composed of odd numbers. Therefore, there are 4500 of them.

c) To construct a 4 digit number consisting of distinct digits we have 9 ways to choose the first digit (we can't pick 0), 9 ways to choose the second (we can't repeat the first) then 8 and 7 ways to choose the last two without repetition. Using the multiplication rule, we find that there are $(9)(9)(8)(7) = 4536$ numbers in the list that have distinct digits.

d) To construct a 4 digit odd number, we have 5 choices for the last digit. Then we have 8 choices for the first (not 0 or same as last digit), 8 choices for the second and 7 choices for the third. Using the multiplication rule again, we get that there are $(8)(8)(7)(5) = 2240$ odd numbers with distinct digits in the list.

e) In each case, we divide by the size of our sample space to compute the probability. Respectively, we get $\frac{4536}{9000} = \frac{63}{125}$ and $\frac{2240}{9000} = \frac{56}{225}$.

6.3.10 - a) The letters in the word THEORY are distinct, so there are $6! = 720$ ways to rearrange them.

b) We can consider 'TH' to be a single letter and find that there are now 5! ways to rearrange the 5 letters. We get another 5! ways when we consider the case for 'HT'. Therefore, in total, there are $(2)(5!) = 240$ such rearrangements.

6.4.16 - a) Since there are 40 boards and 5 are chosen, we get $\binom{40}{5} =$ 658008 combinations.

b) There are 37 boards without a defect so there are $\binom{37}{5} = 435897$ ways to choose 5 boards without any defects. So, all other combinations, must contain at least one defect. Therefore, there are $658008 - 435897 = 222111$ ways to choose 5 boards with at least 1 defective.

6.5.13 - First of all, we can change the question by using the substitution $x_i = y_i - 2$. Then we are looking for the number of solutions to the equation $x_1 + x_2 + x_3 + x_4 = 22$ where $x_i \geq 0$. Following the example done in class, this is equivalent to putting 3 bars with 22 Xs to form a string. This can be done in $\binom{22+3}{22} = 2300$ ways.

6.8.15 - The expected value can be expressed as:

$$E(X) = \frac{10000000}{30000000} + \frac{1000000}{30000000} + \frac{50000}{30000000} + \frac{30000000(-0.6)}{30000000} \doteq -0.2317$$

Therefore, you expect to lose approximately 23 cents on average.

6.9.24 - Since X and Y work independently, the probability they both miss a given error is given by $(0.12)(0.15) = 0.018$. Therefore, if a manuscript contains 1000 errors, we can expect to miss $(0.018)(1000) = 18$ of them.

7.2.26 - a) C is one to one. To prove this, we begin by letting $C(s_1) = C(s_2)$ for two strings s_1 and s_2. Then we get that $as_1 = as_2$ and we conclude that $s_1 = s_2$.

b) C is not onto. For example, there is no string s for which $C(s) = b$.

7.3.19 - Every three digit integer you pick need only contain one digit (repeated 3 times). We cannot, however, choose 000. Also, every three digit integer must contain at least one of the digits from 1 through 9. Therefore, using the pigeonhole principle, there are 9 holes (digits 1 through 9) so to guarantee two numbers have a digit in common, we need to have chosen 10 three digit numbers.

7.4.24 -

$$(g \circ f)(x) = g(f(x)) = g(x+3) = -x-3$$
$$(g \circ f)^{-1}(x) = -(x+3) = -x-3$$
$$g^{-1}(x) = -x$$
$$f^{-1}(x) = x-3$$
$$(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x)) = f^{-1}(-x) = -x-3$$

We see that $(g \circ f)^{-1} = (f^{-1} \circ g^{-1})$.

(7.5.20 and 7.5.21) - There are many possible answers to these questions. Here are a couple examples of each:

One to one, but not onto:

$$f(x) = |x| \cdot x$$
$$g(x) = 3x + 2$$

Onto, but not one to one:

$$h(x) = \left\lfloor \frac{x}{2} \right\rfloor$$
$$k(x) = \begin{cases} \frac{x}{3} & \text{if } x = 3k \text{ for some } k \\ x+1 & \text{otherwise} \end{cases}$$