MATH 3790 - Homework Set 4

Not to be handed in

1. Prove Euler's Formula

2. Prove without the use of Euler's Formula that given a graph G which is a tree, if $|V(G)| = n$ then $|E(G)| = n - 1$.

3. Prove that the n^{th} hypercube is n-connected.

4. Alphonse and Beryl are playing a game. To begin, Alphonse says "January 1st". Then, they alternately name dates such that they change only the month or the day of the month but not both. Also, each date named must be later in the year. The person who names "December 31st" wins. Determine who wins this game and their winning strategy.

5. Peter and Paul play a game with 3 piles of beans labelled A, B and C respectively. On each player's turn, they must remove 1 or 2 beans from pile A, remove 1,2 or 3 beans from pile B or remove 1,2,3 or 4 beans from pile C. As usual, if it is a player's turn and he cannot make a legal move, he loses the game.

a) Assuming the game begins with 3 beans in each pile and Peter goes first, show that Paul has a winning strategy.

b) If the game begins with 7 in pile A, 8 in pile B and 6 in pile C and Peter goes first, determine which player has a winning strategy. Justify your answer.

c) Find the smallest integer $n > 3$ such that if Peter plays first and each pile has n beans, Paul has a winning strategy.

d) Find an expression for all n such that if Peter plays first and each pile has n beans, Paul has a winning strategy.