MATH 3790 - Assignment 1

Due: Sept 23

September 17, 2003

1. Prove that $\sqrt[3]{3}$ is irrational.
2. Prove that $\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$.
3. Given exactly one of each of the Tetris pieces (there are 5 of them), determine if they can be arranged so that they form a rectangle without any overlapping.
4. An ancient puzzle called the *Tower of Hanoi* consists of three pegs on a stand and n punctured discs of different sizes that are placed in decreasing order on one of the pegs. The object of the puzzle is to transfer the pile of discs to another peg, by moving one disc at a time, and without placing any disc on top of a smaller disc. Show that it is possible to solve this puzzle in $2^n - 1$ moves.
5. Do there exists non-zero integers a and b such that one of them is divisible by their sum and the other is divisible by their difference?
6. (BONUS) Prove that e is irrational.