MATH 3790 - Assignment 3

Due Nov 6

October 30, 2003

1. State and prove the AM-GM inequality for 3 terms.

2. Consider the sequence $\{1, \sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, ...\}$. Which term is the largest? Prove your claim.

3. Find all real x, y, z such that

$$-2 < \frac{x^2 + y^2 + z^2}{xy + xz + yz} < 1$$

4.a) How many solutions are there to the equation $a_0 + a_1 + a_2 = 12$ for integers $a_i \geq 1$?
b) How many solutions are there to the equation $a_0 + a_1 + a_2 = 9$ for integers $a_i \geq 0$?

5.a) Prove that the sum of the elements in the n^{th} row of Pascal's triangle is 2^n.
b) Prove that the alternating sum of the elements in the n^{th} row of Pascal's triangle is 0.

6. (Bonus) In a particularly long hallway at a given school, there are exactly 1000 lockers. One day, a bored student walks down the hallway and opens every locker. Then he goes back to the beginning and closes every second locker. He again goes back to the beginning and now opens/closes every third locker (changing it's current state). He continues this process for 1000 passes, each time increasing the space between the lockers he adjusts by one. How many lockers are left open when he is finished?