1. Determine all the values of n such that $n^2 - 19n + 99$ is a perfect square.

2. Determine all n such that $\frac{n^3-n}{24}$ is an integer.

3. Show that n can be written as the sum of two squares if and only if $2n$ can be written as the sum of two squares.

4. Determine the value of $\sqrt{1+\sqrt{1+\sqrt{1+...}}}$

5. Determine the last digit of $183^{217} + 217^{183}$.

6. Suppose that A is a 5-digit number such that when a 1 is put after it (to form a 6 digit number), the result is three times the number formed by putting a 1 before it. Determine the 5-digit number A.

7. Determine all (a, b, c) such that $a^2 + 2b^2 - 2bc = 100$ and $2ab - c^2 = 100$.

8. Determine all integers k for which $k - 2$ divides evenly into $3k$.

9. Let $f(n)$ be the sum of the first n terms of the sequence $0, 1, 1, 2, 2, 3, 3, 4, 4, ...$ Find a formula for $f(n)$ and prove that $f(s+t) - f(s-t) = st$ where s and t are positive integers and $s > t$.

10. If the 7-digit number 2718xy6 is a multiple of 72, with x and y digits, determine all possibilities for the ordered pair (x, y).

11. What are the last two digits of 1!+2!+3!+...+2002!+2003!

12. Prove that if a quadratic equation $ax^2 + bx + c = 0$ has real roots, then a, b and c cannot be consecutive terms of a geometric sequence.

13. Determine all real numbers x that are solutions to both equations $x^2 - kx + 8 = 0$ and $x^2 - 8x + k = 0$.

14. If an integer k is added to each of the numbers 139, 339 and 571, the results are the squares of three consecutive terms of an arithmetic sequence. Determine the value of k.

15. If n is a positive integer, show that $n^4 + 2n^3 + 2n^2 + 2n + 1$ can never be a perfect square.

16. Show that for every positive integer n bigger than 3, $n!$ can be written as the difference of two odd squares.

17. Determine all a such that $x^3 - x + a = 0$ has three integer roots.

18. Suppose that $n! + m! = k!$. If n, m and k are positive integers, determine all possible values for n, m and k.

19. Determine the numerical value of $2004^2 - 2003^2 + 2002^2 - ... + 2^2 - 1^2$.

20. Find a 100-digit number with non-zero digits that is divisible by the sum of its digits.

21. Find all triples (A, B, C) of positive integers such that $A^2 + B - C = 100$ and $A + B^2 - C = 124$.

22. Determine the largest integer k such that 7^k divides evenly into $1^1 2^2 3^3 ... 99^{99} 100^{100}$.

23. In a particular group of horses and chickens, the number of legs is 16 more than twice the number of heads. How many chickens are there?

24. Find all solutions to $x^2 - 5|x| - 6 = 0$.

25. The sum of three positive integers is 50. Taken in pairs, their differences are 3, 4 and 7. Find all possibilities for the three numbers.

26. Show that $n(n + 2)$ is one less than a perfect square. Also, show that $m(m + 1)(m + 2)(m + 3)$ is never a perfect square.

27. Prove that if n is a positive integer which is not a power of 2, then it can be written as the sum of at least two consecutive positive integers.

28. Find a four digit number which is a perfect square with its first two digits equal to each other and its last two digits equal to each other.