MATH 3790 - Test 1

September 24, 2003

1.(a) Do there exist rational numbers a, b such that $a + b$ is irrational?

(b) Do there exist an irrational number a and a rational number b such that $a + b$ is rational?

(c) Do there exist irrational numbers a, b such that $a + b$ is rational?

2.(a) Given an 8x8x8 cube, is it possible to 'tile' it with dominoes of size 2x1x1 if two opposite corners of the cube are removed? (In this case, tiling a 3D object consists of filling interior of the cube).

(b) Given an 8x8x8x8 hypercube, is it possible to 'tile' it with dominoes of size 2x1x1x1 if two opposite corners of the hypercube are removed?

3. Find a closed form expression for the sum:

$$1 + 5 + 9 + ... + (4n - 3)$$

and prove that it is correct **two** different ways.

4. (Bonus) Do there exist two irrational numbers a, b such that a^b is rational?