MATH 3790 - Test 2

October 22, 2003

1. Determine the last digit of $123^{321} + 321^{123}$.

2. Find all pairs of integers (a, b) such that $ab^2 - a - b^2 = 4$.

3. Let a, b, c be odd integers. Prove that $ax^2 + bx + c$ has no rational roots.

4. Consider the equation $x^2 + y^2 = z^2 + 2$.
a) Prove that if the relationship is true, z must be even.
b) Prove that if $z = y + 1$ then there are infinitely many solutions.
c) Prove that if $z = y + 5$ then there are no solutions.

5. Given the set $\{\pi, 2\pi, 3\pi, ..., 999\pi\}$, prove that at least one of these numbers is within $\frac{1}{1000}$ of an integer.

6. (Bonus) Find an 11-digit number which is divisible by 792 and whose digits are all from the set $\{3, 4, 5, 9\}$.