MATH 3790 - Test 3

November 13, 2003

1. a) Prove that $a^2 + a + b^2 + b \geq -\frac{1}{2}$.
b) Find one pair (a, b) where this holds with equality.

2. Assume that the cubic $x^3 + bx^2 + cx + d$ has 3 real roots. Prove that $b^2 \geq 2c$.

3. For each of the following 3 equations, find a positive integral solution or prove that none exist.
a) $x^2 - 3y^2 = 0$
b) $x^2 - 3y^2 = 1$
c) $x^2 - 3y^2 = 3$

4. In a particular game show, one game involves having a disc fall through a triangular shaped array of pins. There are six levels of pins, and at each level the disc has an equal probability of falling to the right or to the left of the pin.
a) How many different paths are there such that the disc lands in the center at the bottom of the array?
b) What is the probability that the disc ends up in the center at the bottom of the array?

5. Four couples (including Mr. and Mrs. Brown) attended a party. During the course of the party, some people shook hands. No one shook their own hand and no one shook the hand of their spouse. At the conclusion of the party, Mrs. Brown asked the other seven guests how many hands they shook and discovered that they all shook a different number of hands. How many hands did Mr. Brown shake?

6. (Bonus) If a, b, c are the sides of a triangle, show that

$$(a + b)(b + c)(a + c) \geq 8abc$$