MATH 3790 - Test 4

December 4, 2003

1. Show that given a forest with n vertices and m edges, the number of components is $n - m$.

2. Suppose u, v, x, y are vertices in a connected graph G. There is a path P_1 of length k from u to v, and a path P_2 of length k from x to y. Also, there are no paths in G which have length greater than k. Show that P_1 and P_2 have at least one common vertex.

3. (a) Prove that if $|V(G)|$ is odd, then $\exists v \in V(G)$ with $d(v)$ even.

Consider a game played by two players, Left and Right. The game begins with a graph G. The players alternately choose a vertex and delete it and all incident edges from G.

(b) If both players can only remove even degree vertices, find all graphs where the first player has a winning strategy.
(c) If Left can only remove even degree vertices and Right can only remove odd degree vertices, show that given a graph G if Left can choose who plays first he can always win.

4. Alphonse and Beryl are playing a game with a pile of toothpicks. Starting with Alphonse, they alternately remove 1, 2, 4 or 8 from the pile. The person who takes the last toothpick wins. Find an expression for all n such that if the game starts with n toothpicks, Beryl has a winning strategy.

5. Imagine a Pascal-like triangle where each row begins with a 1 but ends with a 2. All other values are calculated as the sum of the numbers above and to the right and left. What is the sum of the numbers in the 13th row?

1 2
1 3 2
1 4 5 2
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