Assignment 2 - Due Wednesday Januar 21

(1) Express each of the following numbers as a product of primes: 6, 24, 27, 35, 120.

(2) Does a nonprime divided by a nonprime ever result in a prime? Does it ever result in a nonprime? Always? Sometimes? Never? Explain your answers.

(3) **For Extra Credit** Consider the following sequence of natural numbers: 1111, 11111, 111111, 1111111, 11111111, ... Are all these numbers prime? If not, can you describe infinitely many of these numbers that are definitely not prime?

(4) Suppose a certain number when divided by 13 yields a remainder of 7. If we add 22 to our original number, what is the remainder when this new number is divided by 13?

(5) In this exercise and the next one we want to compare the number of perfect squares with the number of primes. How many perfect squares are less than or equal to 36? How many are less than or equal to 144? In general, how many perfect squares are less than or equal to n^2? Using these answers, estimate the number of perfect squares less than or equal to N (for any number N). [Hint: your estimate may involve square roots and should give the exact answer when N is itself a perfect square.]

(6) Using a calculator or computer, fill in the last two columns of the following chart:

n	Number of primes up to n	Appr. number of squares up to n	(number of primes)/ (number of squares)
10	4		
100	25		
1000	168		
10,000	1229		
100,000	9592		
1,000,000	78,498		
10,000,000	664,579		
100,000,000	5,761,455		
1,000,000,000	50,847,534		

Given the information you have found, what do you conclude about the proportion of prime numbers to perfect squares? Are prime numbers more common than perfect squares or less common?

For Extra Credit: Use the prime number theorem to conjecture a formula for the quotient of the number of primes up to n divided by the number of squares up to n.