Assignment 5 - Due Friday March 6

(1) **Discovering Fibonacci Relationships** By experimenting with numerous examples in search of a pattern, determine a simple formula for $(F_{n+1})^2 + (F_2)^2$ – that is, a formula for the sum of the squares of two consecutive Fibonacci numbers.

(2) **A New Start** Suppose we build a sequence of numebrs using the method of adding the previous two numbers to build the next one (just as for the Fibonacci sequence). This time, however, suppose our first two numbers are 2 and 1. Generate the first 15 terms. This sequence is called the *Lucas sequence* and is written as L_1, L_2, $L_3, \ldots$ Compute the quotients of consecutive terms of the Lucas sequence as we did with the Fibonacci numbers. What number do these quotients approach?

What role do the initial values play in determining what number the quotients approach? In order to get an answer to this question, try two other first terms and generate a sequence. What do the quotients approach? Can you draw any conclusions?

(3) **Digging Up Fibonacci Roots** Using the square root key on a calculator, evaluate each number in the top row and record the answer in the bottom row of the following table:

Number	$\sqrt{\left(\frac{F_3}{F_1}\right)}$	$\sqrt{\left(\frac{F_4}{F_2}\right)}$	$\sqrt{\left(\frac{F_5}{F_3}\right)}$	$\sqrt{\left(\frac{F_6}{F_4}\right)}$	$\sqrt{\left(\frac{F_7}{F_5}\right)}$	$\sqrt{\left(\frac{F_8}{F_6}\right)}$	$\sqrt{\left(\frac{F_9}{F_7}\right)}$
Computed Value							

Looking at the chart, make a guess as to what special number $\sqrt{F_{n+2}/F_n}$ approaches as n gets larger and larger.

Extra Credit Explain the result you found in the previous part.

(4) **Flower Heads** Suppose that a flower head makes a .2 clockwise turn before releasing its next seed. Draw the shape of the seed head after 20 seeds have been released. Do the same for a flowerhead that makes a .21 clockwise turn, and for a flowerhead that makes a .19 clockwise turn.

(5) **Golden Rectangles**

(a) Explain what makes a rectangle a *Golden Rectangle*.

(b) Suppose that you have a golden rectangle and tehn you attach a square along its longer side to create a new rectangle. Is this new rectangle again golden? What if we repreat the process with the new larger rectangle? (For an image, see Page 245 in the book.)

(6) **Counterfeit Gold?** Draw a rectangle with its longer edge as the base (it could be a square, it could be a long and skinny rectangle, whatever you like, but take something that is not close to a Golden Rectangle). Now, using the top edge of rge rectangle, draw the square just above the rectangle so that the square's base is the top edge of the rectangle. You have now produced a large new rectangle (the origional rectangle togetehr with the square sitting above it). Now attach a square to the right of this rectangle so that the square's left side is the right edge of the large rectangle. You've constructed an even larger rectangle. (for some images to guide you, see Page 246 of the book.)

Repreat this procedure - add a square to the top of the new rectangle, then add a square to the right, etc. Start with a small rectangle and conintue the process until you have almost filled the page. Now measure the dimensions of your final rectangle. What is the ratio of the side lengths? How does it compare to the Golden Ratio? Experiemnt with various starting rectangles. What do you notice about the ratios?