Projects for MATH 1600 - Fall 2008

General requirements: you may choose how you submit the results of your project. You can either write them up nicely and submit a written report, or you can give a presentation in class (for up to 25 minutes). The due date for the written report is the last day of classes, the due date for a presentation would be some time in the last two weeks of classes.

Please choose your project topic by November 12 and let me know during the next week whether you would like to submit a written project or give a presentation. You may also choose to work with two people on a project. Some projects are really intended for two people, so discuss with me how much you need to do.

- **Wall Paper Patterns** Suppose that you want to create a wall paper pattern that repeats itself over and over again so that you can make it as long and as high as you want. You could just create a nice picture and then repeat it over and over again by translation, but you might like to add some rotation or reflection symmetry. What are your possibilities? It turns out that there are only 17 possible ways to do this! For this project you will have a chance to both create new patterns and to look at existing tesselations and determine their symmetry.
- **Polygonal Curvature** In this project you will construct and study 2-dimensional objects (surfaces) made out of polygonal tiles. Just as for polyhedra we require that the intersection of any two tiles is an edge or a vertex or the empty set. We will now look at the sum $s(v)$ of the angles of the tiles meeting at a vertex v. We define the *polygonal curvature* as $k(v) = 2\pi - s(v)$. If $k(v) = 0$ the tiles around v fit together in a flat way, as in the plane. If $k(v) \neq 0$, the surface is curved, as is the case for all the polyhedra we have considered. So it makes sense to call this the curvature. In this project you will learn more about the curvature of some of the objects we have studied already, and you will also construct a new surface for which the curvature is negative! You will then look at what the effect of the curvature is on the area and the sum of the angles of a triangle.
- **Euler's Formula for the Surface of a Donut** Suppose that you were a small insect living on the surface of a donut. How would you be able to find out that the shape of your world is that of a donut surface and not the surface of a Timbit without flying away to consider your world from a distance? After you have considered this problem, you will find out some interesting results about graphs on the surface of a torus: you can draw K_5 on the surface of a torus without crossing any edges! What does this mean for Euler's formula?
- **Knots** You may know from experience how hard it can be to untangle a knot in real life. And when you try to do it in theory it can be even harder... Knots are 1-dimensional objects that live in 3-dimensional space, so when we draw them we always have a projection of the knot. So deciding whether two knots are the same is very hard. There is not a general algorithm that will do this for us. in this kind of situation mathematicians come up with notions of *invariants*. They find ways of attaching a number (or some other mathematical object, such as a polynomial) to each knot in such a way that if the knots are really the same they will get the same number or object. Invariants don't completely solve the problem for us: there may

be knots that have the same invariant, but are not the same knot. So we always keep looking for new invariants.

In this project you will first learn some moves that you can use to "untangle" knots on paper (they are called Reidemeister moves). And then you will study a couple of different knot invariants and show how they help us to distinguish between different kinds of knots.

You may also look at some of the applications of knots in genetics and medicine.

- **Fibonacci and Lucas Sequences** Both the Fibonacci and Lucas numbers satisfy the recurrence relation $u_{n+2} = u_{n+1} + u_n$. The Fibonacci numbers start with $F_0 = 0$ and $F_1 = 1$, where the Lucas numbers start with $L_0 = 2$ and $L_1 = 1$. In this project you will study the divisibility properties of these numbers. For example, for which values of l and m do we have that $F_l | F_m$? For any given number q is there always a Fibonacci number F_m such that $q|F_m$, and is there always a Lucas number L_m such that $q|L_m$?
- **Algebraic Infinitesimals** The algebraic infinitesimals form an extension of the real numbers where we have added a new number, called ε, such that $\varepsilon^2 = 0$, but $\varepsilon \neq 0$. So the equation $x^2 = 0$ has now two distinct roots: $x = 0$ or $x = \varepsilon$. You can show that all numbers in this new number system are of the form $a + b\varepsilon$ for real numbers a and b. Just as for the complex numbers, we can represent these numbers by vectors in the plane. In this project you will explore some of the structure of these numbers and you will find out why they are good for taking derivatives of polynomials.
- **Fractals** Fractals can be created in various ways, but they all share the property that in some sense they remain equally complicated no matter how much you zoom in.

  One way to create fractals is by the use of an iterated function system - you take a finite set of contracting transformations $F_1, F_2, \ldots, F_n$ of the plane and any point p. Then you look at the sets $P_0 = \{p\}$, $P_1 = \bigcup_{i=1}^n F_i(P_0)$, $P_2 = \bigcup_{i=1}^n F_i(P_1)$, ... It turns out that the limit $\lim_{n\to\infty} P_n$ does not depend on the initial point p! In this project you get to prove this result and look at various other properties of fractals, such as their dimension (they usually have a dimension that is not an integer). You may also look at families of fractals and how they develop as you change one of their parameters. Or you may look at what happens if you add some isometries such as reflections, or even inversions, to your set of functions. Another neat property of this type of fractals is that you can make approximations of any object you like and you can make the distance between your fractal and the chosen object less than ε for any $\varepsilon > 0$ by using more transformation in the iterated function system.